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Training Program for the Development of Teaching Competencies of Panamanian Elementary School Teachers for Teaching Mathematics

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Abstract

Reflection on the training needed by Panamanian teachers has little research tradition, although recently, training programs have been developed for teachers to teach mathematics in accordance with current trends in the teaching of this subject. This article explains one of these postgraduate training programs. Specifically, the guidelines for its design and some results of its second implementation are discussed. Among the most important results for this article we have a) the creation of the platform as well as the set of assignments designed, which have been used for the training of subsequent promotions of teachers; b) It highlights the change in the criteria that guide the teacher's practice at the end of the program, in relation to the criteria used before the training, particularly emphasizing the relevance acquired by the emotional aspects and those related to interaction in this process.

Introduction

Debating teacher training, although a topical issue, is not a simple task since it is closely related to social, political and cultural aspects. Scientific institutions in several countries, especially in Latin America, have discussed teacher training as a consequence, above all, of the need to seek higher quality teaching and, in particular, to obtain better results in student learning and in PISA-type assessment tests. The different studies that have been conducted consider that the problem of teacher training is a complex issue that requires a multidisciplinary analysis due to the dynamics of societies in constant change and transformation. Rosas (2000) in this sense mentions the challenges of teacher training and states that it is not a simple endeavor since it must be closely associated with social changes as well as with the needs that arise from the vertiginous changes that are currently being experienced.

There are several theoretical models and numerous studies in the field of Mathematics Education that characterize the knowledge and competencies that a mathematics teacher should have to perform their teaching task (Ball et al., 2008; Ball & Bass, 2009; Blömeke et al., 2014; Carrillo-Yañez et al., 2018; Godino et al., 2017; Hill et al., 2008; Llinares, 2003; Pinheiro & Gusmão, 2022; Pimenta, 2005; Pino-Fan et al, 2022; Pino-Fan et al, 2013; Pinto, 2017; Shulman, 1987). These models and studies coincide in that it is not enough to know mathematics to teach it well, and that it is necessary, among other aspects, didactic, pedagogical, and affective competencies and knowledge. On the other hand, they also agree that the required teacher training to be a mathematics teacher is not only obtained from contact with practice, but also requires knowledge and skills to understand and reflect on this practice.

Furthermore, as Rozada (2007) explains, there is still an absence of systematic theorizations on teacher

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training in this country, which is related to the process of knowledge transfer as well as the implementation of such knowledge in the schools; On the other hand, an element that influences this problem is the fact that teachers do not actively participate in the public agenda associated with education; as a result, they are not very active and immersed in the discussions about education and this leaves them in the position of actors in an educational system that is partially defragmented.

The mathematics education needed by Panamanian teachers raises, among others, the following research questions: What are the professional competencies that mathematics teachers need to develop? How to develop and assess them? The answer to these questions, in turn, is related to the answer to the following more general question: What are the professional competencies needed by primary school teachers to teach mathematics? Our research group – Research Group in Mathematic Education of University of Panama (GIEM21) - has been interested, in the framework of different projects, in aspects related to these questions. In particular, it has been particularly involved in the development of training programs to ensure that Panamanian teachers have the necessary knowledge and skills to teach mathematics in accordance with current trends in the teaching of this subject. The aim of this article is to describe the Diploma in Didactic Strategies for Mathematics Education (DSEE), which is a postgraduate training program, and, to delve into the design of this diploma along with a discussion of the results of its second implementation in a semi-virtual format.

Once this part of the pedagogical and methodological structure of the program has been explained, it is intended to reflect on the competencies needed by the Panamanian elementary school teacher to explain mathematics according to the Ministry of Education of this country, as well as the universities that train teachers. Likewise, the theoretical framework that has served to reinterpret the competencies of the first section is explained, thus giving theoretical support to the design of the EDEM diploma program: the theoretical model of Mathematics Didactic-Mathematics Knowledge and Competencies of the Mathematics teacher (CCDM model). On the other hand, the guidelines for the design of the DTSD diploma course and some aspects of its implementation are explained.

1.2. Competencies Required by Primary School Teachers in Panama.

According to the Ministry of Education, the continuous training of active teachers in Panama requires an autonomous teacher, capable of responding to the demands and requirements posed by a society in constant change due to the advances in mathematical, technological, and social knowledge, competencies related to those that the World Economic Forum (Whiting, 2020) has set forth in the ten job skills of tomorrow. Thus, the continuing education offered by the Ministry of Education of Panama is intended to be a process of permanent updating that enables teachers to perform their pedagogical and professional practice in a meaningful and innovative way. For this reason, in the University of Panama's offer of continuing education in Mathematics and Pedagogy for primary and preschool teachers in the period 2013-2018, the teacher is conceived as a professional trained (suitable) to build mathematical knowledge and apply new methodologies, based on the experience faced in the classroom.

However, contradictorily to the principles just discussed above, in the last six years there is an imbalance in the initial training of teachers, since the following content blocks have different emphasis in their studies:

1. General Didactics (a very strengthened area in the programmatic contents of studies of the bachelor's degree in primary and preschool education).
2. Specialized Didactics of the discipline (area with little emphasis in the programmatic contents).
3. Mathematical Knowledge (area very weakened in the programmatic contents of studies).

Despite the official national consensus that initial and ongoing teacher training should be a first-rate quality component of the education system, teacher training centers in Panama, both official and private, continue to reproduce the traditional school culture and, in turn, many aspiring educators arrive with equally traditional schooling paths.

On the other hand, the reality of continuing education is not always consistent with this consensus on how teacher training should be, which can be observed, firstly, in the school-administrative calendar where there is no space for training within the working day of teachers. Meanwhile, there is a system that facilitates, through regulation No. 5261 that approves the table of affinities of the academic degrees required to aspire to teaching positions in the Ministry of Education (available at: https://www.gacetaoficial.gob.pa/pdfTemp/29421/88453.pdf), a list of degrees that do not require relevant mathematical training to be a teacher.

The United Nations Development Program (UNDP), supported by the Presidency in the Dialogue Table "National Commitment to Education" (2017), presents the following competencies needed in the profile of the primary school teacher, considered by this Program a national priority.

1. Possess a solid Mathematical training, linked to science and technology, and interested in solving pedagogical-didactic problems of teaching at different levels.
2. To have access to mathematical knowledge without biasing knowledge, with scientific, humanistic, social, aesthetic, and ethical interest.
3. To have permanent improvement and updating and the possibility of enriching their academic background through transdisciplinary contact.
4. To have knowledge of Mathematics and its applications for a better development of teaching.
5. Possess the ability to produce educational material using different technologies.
6. Demonstrate a reflective attitude and ability to collaborate in educational research related to the field of mathematics education.
7. Have the capacity for critical analysis of pedagogical and socio-cultural problems generated in the educational community.

In order to reduce the gap between the necessary professional competencies and to be able to follow the new curricular orientations as well as the reality of initial and continuous teacher training, it is required the design and implementation of training programs oriented to the development of the new professional competencies needed by teachers. In this line, our research group GIEM21, within the framework of different Reaserch + Development projects financed by SENACYT, has designed a teacher training program, which began with the Diploma in Didactic Strategies for Teaching Mathematics (EDEM), whose objective is the development of these professional competencies. For this purpose, these competencies have been reinterpreted in terms of the competencies of the CCDM model and, in particular, in the sub-competency of assessing the didactic suitability of instructional processes.

1.3. Didactic-Mathematics Knowledge and Competencies Model for the Mathematics Teacher (CCDM Model).

In the framework of the EOS (Godino et al, 2007 and 2019) a theoretical model of Didactic-Mathematics Knowledge and Competencies of the mathematics teacher (CCDM model) has been developed (Breda et al, 2017; Godino et al, 2017; Pino-Fan et al, 2022)

1.3.1 The Notion of Competency and Key Competencias

The competency in the CCDM model is understood from the perspective of competent action, considering it as the set of knowledge, skills, affective dispositions for action, tools for reflection, etc. that allow the effective performance in the contexts of the profession of the actions mentioned in its formulation. It is a potentiality that is actualized in the performance of effective (competent) actions. According to Font et al (2015), it was considered that the resolution of a task is the starting point for the development and/or evaluation of a teacher's competence, the above by virtue of the fact that the task produces the perception of a professional problem to be solved, for which the teacher (or future teacher) mobilizes skills, knowledge and attitudes, to perform a practice that attempts to solve the problem. On the other hand, it is to be expected that such practice is performed with more or less success (achievement), which is considered evidence that the person can perform practices similar to those described by one of the indicators of the competency, which in turn is associated with a certain level of development of the competency. This formulation of the competency, in order to be operational, needs a characterization of its development (levels of development and indicators) as proposed in Pino-Fan et al (2022).

The subject of competency development in higher education has acquired great relevance. In general terms, competencies can be defined as the ability to apply knowledge, skills and values in specific situations to achieve specific objectives (McLagan, 1997). The interest of Higher Education Institutions in this topic lies in their role of training the future of the labor force, since competencies are fundamental in professional training as they affect the development of practical skills applicable in the labor world (Alfaro-Ponce et al. 2023). The issue of competencies has become a nodal point for the labor market as it is becoming increasingly competitive and dynamic and therefore employers are looking for candidates who can demonstrate concrete and relevant skills for their work (Oyarce et al., 2020). Therefore, the CCDM model is considered that the two key competencies of the mathematics teacher are mathematical competence and didactic analysis and intervention competence, being the fundamental core of the latter (Breda et al, 2017): designing, applying, and assessing own and others' learning sequences, through didactic analysis techniques and quality criteria, to establish cycles of planning, implementation, assessment and raise proposals for improvement.

1.3.2 Characterization of the Competency of Didactic Analysis and Intervention.

This general competency is composed of different sub-competencies (Breda et al, 2017): 1) sub-competency of analysis of mathematical activity--this sub-competency, in Godino et al (2017), is further decomposed into two: global meaning analysis competency and ontosemiotic analysis competency of mathematical practices--; 2) sub-competency of analysis and management of interaction and its effect on student learning; 3) sub-competency of analysis of norms and metanorms; and 4) sub-competency of assessment of the didactic appropriateness of instructional processes.

Sub-competency in the analysis of mathematical activity.

In the area of mathematics education there is no paradigm that indicates how the analysis of mathematical activity should be conducted. From the CCDM model it is assumed that the theoretical tools of EOS allow such analysis in terms of mathematical practices, objects, and processes. The teacher's identification of the objects and processes involved in mathematical practices makes it possible to understand the progression of learning and to evaluate students' mathematical competencies.

Sub-competency of analysis and management of didactic configurations and their effect on learning.

The notion of didactic configuration has been introduced in EOS as a tool for the analysis of interactions in instructional processes (Godino et al, 2006). It is a theoretical construct for modeling the articulation of teacher and student actions around a given task and content (a configuration of primary objects and processes) of teaching and learning, where knowledge emerges from the interaction process itself. The mathematics teacher must have competence in the design and management of didactic configurations in order to ensure student learning in a given instructional process.

Normative analysis sub-competency

Teaching and learning processes are supported by, and are dependent on, a complex network of norms and meta-norms of different origin and nature (Godino, Font, Wilhelmi & Castro, 2009), whose explicit recognition is necessary to be able to understand the development of instructional processes and guide them towards optimal levels of appropriateness. The mathematics teacher has to develop the competence of normative analysis of mathematical instructional processes in order to answer questions such as the following: What norms condition the development of instructional processes? Who, how and when are the norms established? Which and how can they be changed to improve learning? and so on.

Sub-competency of assessment of the didactic suitability of instructional processes.

EOS proposes the notion of didactic appropriateness as an essential tool for the assessment of instructional processes. Fixed on a specific topic in a given educational context, the notion of didactic suitability (Breda et al, 2018; Godino, 2013) leads to being able to answer questions such as: What is the degree of didactic suitability of the teaching-learning process implemented? What changes should be introduced in the design and implementation of the instructional process to increase its didactic suitability in future implementations?

The didactic suitability of an instructional process is defined as the degree to which such process (or a part of it) meets certain characteristics that allow it to be qualified as optimal or adequate to achieve the adaptation between the personal meanings achieved by the students (learning) and the intended or implemented institutional meanings (teaching), taking into account the circumstances and available resources (environment). An instructional process will achieve a high degree of didactic suitability if it is able to articulate in a coherent and systemic way, the following six partial criteria of suitability, referring to each of the six facets involved in this process:

1. Epistemic suitability, which refers to the mathematics taught being "good mathematics". To achieve this, in addition to taking the prescribed curriculum as a reference, it is a matter of taking as a reference the institutional mathematics that has been transposed into the curriculum.
2. Cognitive suitability, which expresses the degree to which the intended/implemented learning is in the students' zone of potential development, as well as the proximity of the learning achieved to the intended/implemented learning.
3. Interactional appropriateness, degree to which the modes of interaction allow the identification and resolution of conflicts of meaning and favor autonomy in learning.
4. Mediational suitability, degree of availability and adequacy of the material and temporal resources necessary for the development of the teaching-learning process.
5. Affective adequacy, degree of involvement (interest, motivation) of the students in the study process.
6. Ecological suitability, degree of adaptation of the study process to the center's educational project, the curricular guidelines, the social environment, etc.
7. For each of these criteria, a system of components and associated indicators that can be assessed on a scale is proposed. This is a rubric system that allows a complete and balanced assessment (or self-assessment) of the elements that, as a whole, make up a suitable instructional process in the area of mathematics.

1.3.3 Knowledge of the Mathematics Teacher

In order to develop the required professional competencies in the training processes, teacher trainers must analyze the teaching practices that teachers perform for the resolution of proposed professional tasks, as well as make use of the didactic-mathematical knowledge activated in these tasks, thus being able to find indicators that justify the assignment of degrees of development of the professional competency to be evaluated.

There are several models regarding the knowledge that a mathematics teacher should have to adequately manage student learning (e.g., Hill, Ball, & Schilling, 2008; Rowland, Huckstep, & Thwaites, 2005; Schoenfeld & Kilpatrick, 2008). In Pino-Fan, Assis and Castro (2015) a model is proposed to characterize the Didactic-Mathematical Knowledge (CDM) of teachers, which considers, among other aspects, the contributions and developments of the various models of mathematics teacher knowledge, and the theoretical and methodological developments of EOS. Therefore, the CDM model (a part of the CCDM model) suggests that the teacher's knowledge is organized in three main dimensions: mathematical, didactic and meta-didactic-mathematical.

The first mathematical dimension refers to the knowledge that allows teachers to solve mathematical problems or tasks specific to the educational level at which they will teach (common knowledge), as well as to establish connections between the mathematical objects of that educational level and mathematical objects that will be studied at later levels (extended knowledge). According to the authors of the various models of mathematics teacher knowledge, the second dimension, in addition to the mathematical content, refers tha the teacher must have knowledge of the various factors that influence the planning and implementation of the teaching of this mathematical content. Consequently, the didactic dimension of the CDM proposes six subcategories of teacher knowledge:

1. Epistemic facet, which refers to specialized knowledge of the mathematical dimension (use of diverse representations, arguments, problem-solving strategies and partial meanings of a mathematical object), and incorporates notions such as knowing mathematics in depth and breadth (Schoenfeld and Kilpatrick, 2008) and "specialized content knowledge" (Hill, Ball and Schilling, 2008).
2. Cognitive facet, which refers to knowledge about students' cognitive aspects (difficulties, errors, conflicts, learning, etc.).
3. Affective facet, which refers to knowledge about the affective, emotional and attitudinal aspects of students.
4. Interactional facet, knowledge about the interactions that take place in the classroom (teacher-students, student-student, student-resources, etc.).
5. Mediational aspect, knowledge about the resources and means that can enhance students' learning, and about the time allotted for teaching.
6. Ecological aspect, knowledge about curricular, contextual, social, political, economic aspects, etc., that influence the management of student learning.

The third dimension of CDM, the metadidactic dimension, is the knowledge necessary to reflect on one's own practice (Schön, 1983; Schoenfeld and Kilpatrick, 2008), which allows the teacher to evaluate the instructional process and to redesign it in order to improve it in future implementations. The three dimensions described above are present in the different phases of the instructional process of a given mathematical content: preliminary study, planning, implementation and assessment (Pino-Fan et al, 2018).

1.3.4. Reinterpretation of the Competencies of the Panamanian Elementary School Teacher in the CCDM Model.

The seven competencies proposed by the United Nations Development Programme (UNDP), with the support of the Presidency in the Dialogue Table "National Commitment to Education" (2017), discussed at the end of section 1 can be reinterpreted in terms of the competencies of the CCDM model. In particular, if we limit ourselves to the two key competencies of said model, the first and fourth clearly relate to mathematical competence, while the last three (fifth, sixth and seventh) do so with the competence of analysis and didactic intervention, while the second and third relate, in turn, to mathematical competence and to the competence of assessment of didactic appropriateness. Thus, the fact of reinterpreting the competencies proposed by the UNDP in terms of the competencies of the CCDM model allowed us to elaborate a proposal of competencies for primary school teachers in Panama with theoretical support in the area of Mathematics Education, coherent with the country's educational guidelines, and also to design and implement training programs for their development.

2. Matherial and Methods: Design and Implementation of EDEM

This section explains the design guidelines and some aspects of DTS implementation.

2.1 Competencies of the Diploma in Didactic Strategies for the Teaching of Mathematics (EDEM)

The Diploma in Didactic Strategies for Mathematics Education (EDEM) is a teacher training project for practicing elementary school teachers and graduate students of the Faculty of Education Sciences of the University of Panama (FACE). It was designed for the development of the competencies proposed by the UNDP (in particular, its reinterpretation in terms of the CCDM model) and with the financial support of the Vice Rector's Offices of Research and Graduate Studies and Extension of the University of Panama. It was also endorsed by the Ministry of Education (MEDUCA) and received financial support from the Inter-American Development Bank (IDB) and the National Secretariat of Science, Technology and Innovation (SENACYT).

Previously, a pilot EDEM diploma course was designed and implemented from March to June 2018, in which a multiple case study methodology was used with nine teachers from the Omar Torrijos Elementary School, located in Panama Centro. With the approval of the school authorities and the Ministry of Education and FACE, the participants carried out a wide range of pedagogical activities and mathematical tasks involving the active use of technology in a blended learning modality (Morales-Maure, 2019).

As part of the pilot study, a content analysis was conducted of the graduate profiles of two of the main higher education institutions offering teacher training in Panama, as well as the teacher competencies established by MEDUCA in its guiding documents, with the objective of seeking concordance between these competencies and the competencies that would later be developed with the EDEM. The post-pilot editions of the EDEM have been designed and implemented taking into account the strengths and limitations identified in the pilot project.

2.2. Objectives of the EDEM Diploma Course

The Diploma in Didactic Strategies for Teaching Mathematics is aimed at training preschool and elementary school teachers, and its general objective is to provide a solid professional training, which allows teaching mathematical content in a suitable way with an updated methodology, considering the context in which the teacher works. This general objective is specified in the following specific objectives:

1. To train the teacher according to the new trends in Mathematics Education on the teaching and learning of mathematics.
2. To develop in the teacher the competences and knowledge necessary for the teaching of mathematics in primary education;
3. To provide teachers with criteria for the design, implementation, evaluation and improvement of didactic sequences for the teaching and learning of mathematics.

2.3. Structure of the EDEM

To achieve its objectives, this diploma course, in its first two editions, was structured in four modules - Module 1: Introduction to the didactics of primary education in Mathematics; Module 2: Educational Mathematics and its Didactics I; Module 3: Mathematics and its Didactics II and Module 4: Intervention and reflection on one's own practice -, with a four-month duration in blended learning mode (170 virtual hours and 34 classroom hours) and with the participation of international and Panamanian teachers with training in mathematics and its didactics. Subsequently, two more editions have been held, so far, in virtual format and with a fifth module - Module V: Communicative skills for the management of the mathematics class and for collaborative work - (with a length of 320 hours).

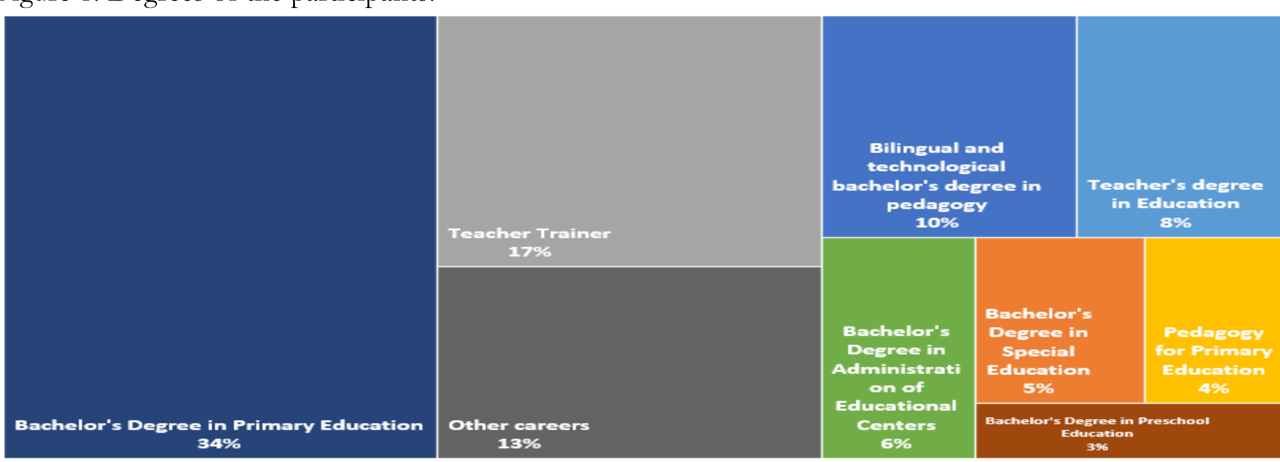
To implement the diploma course, a virtual classroom was created on the interactive technological platform Moodle (Virtual Academic Support Environment) developed by the Virtual Campus of the University of Panama, whose link is located on the University's website. Participants had permanent access to the virtual sessions throughout the course. The most frequent attendance was at night, due to their higher availability in this time slot.

2.4. Participants

As an example of the type of participants, below is the information about the participants in the second edition of the EDEM Diploma. There were 150 primary school teachers from six educational regions, namely: Coclé, Colón, Azuero, Veraguas, Central Panama and West; of which 51% were teachers graduated from the University of Panama, and 17% were normal teachers. Given that 5% of the participating teachers did not have a bachelor's degree, taking this diploma could be a way to help them obtain it (see Figure 1).

It is important to point out that 13% of the participants (other careers), although they were practicing teachers, did not have an academic pedagogical background (they had training related to the education profile). This data is relevant since it shows that a not insignificant percentage of teachers have not had specific training as teachers.

Figure 1: Degrees of the participants.



3. Results and Discussions

3.1 About the Competencies Developed by EDEM

As part of the pilot study, a content analysis (thematic) of the graduate profiles of two of the main higher education institutions that offer teacher training in Panama, and also of the teaching competencies established by MEDUCA in its guiding documents, was conducted in order to seek concordances between these competencies and the competencies that would later be developed with the EDEM. As previous categories, the generic and specific competencies proposed in Font et al (2012) and in the CCDM model (Pino-Fan et al, 2022) were used.

As a result of this thematic analysis, the following cross-cutting and specific competencies (themes) were obtained from the codes found in these documents (Tables 1 and 2):

Table 1: Thematic Matrix: Generic or cross-cutting competencies of the primary school teacher in Panama

	Professional know-how. Citizenship	**Comunication**	**Learning to learn/ Organize continuous training**	**Relationship with families**
Article 331- Education Organic Law No. 60 year, 2004	Participates in the life of their school community. Professional pride. Teamwork skills. Leadership. Develop national and universal values in the student. Noble character.	Verbal and non-verbal communication skills. Leadership.	Self-critical. Permanently updated. Teamwork skills. To know and correspond to the professional and intellectual area.	Participates in the life of its school community.
University of Panama	Considers cross-cutting themes or axes and their relationship with values, giving importance to citizenship training and life in democracy. Strengthens the development of human potential. Critically analyzes educational policies for their historical relationship in the national, regional, Latin American and international spheres.	Apply oral, written and gestural communication efficiently.		
UNACHI	Educates in citizenship and democracy. Educate in values			

Source: own elaboration

Table 2: Thematic Matrix: Specific competencies of the primary school teacher in Panama

	Mathematical Competency	**Competency in didactic analysis and intervention**	**Innovation and research initiation**
Article 331- Education Organic Law No. 60 year, 2004	Mastery of the content taught. Knowledge of and correspond to the professional and intellectual area.	Mastery of methodological techniques. Versatility in practice. Know-how. Pedagogical skills.	
University of Panama		Promotes the development of logical, mathematical, creative and critical thinking of students in single-grade and multi-grade educational centers, informal and non-formal continuing education systems. Considers the transversal themes or axes and their relationship with values, giving importance to citizenship training and life in democracy. Reinforces the development of human potential. Master the theoretical-methodological foundations that support an integral evaluation. Master the theoretical framework of the curricular approach in the didactics of mathematics. Plans and guides the teaching-learning process. Plans the teaching-learning process according to educational needs.	Diagnose the reality of the educational environment to generate projects with participatory and innovative approaches. SWOT. Researches, shares and applies the results to systematically transform educational practices.
UNACHI		Master the discipline's knowledge	

Source: own elaboration

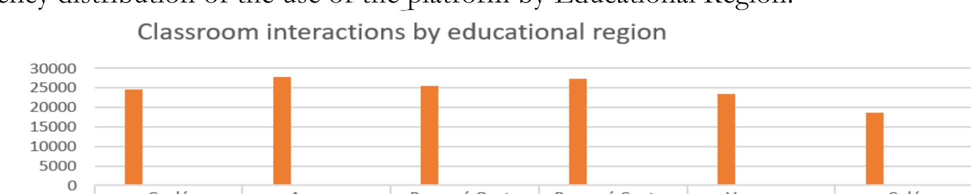
These results allow us to conclude that a diploma course such as the EDEM, which has been designed to ensure that participants develop the competencies described in Fon et al. (2012) and those contemplated in the CCDM model, develops the generic and specific competencies of the graduate profiles of two of the main institutions of higher education that offer teacher training in Panama, and also the competencies of teachers established by MEDUCA in its guiding documents; in addition to developing the competencies proposed by the UNDP.

3.2 Teachers Expectations

At the beginning of the course (module 1), the participants of the second edition of the EDEM expressed their expectations about the course. These expectations showed a great interest in professional improvement, motivation and the need for training in the didactics of mathematics. The teachers also expected a contribution of effective strategies for teaching mathematics, with a strong affective component, i.e. to make their students enjoy mathematics, the learning of information technologies and the learning of innovative strategies. We identified teachers with great commitment and interest.
The teachers' expectations are coherent with the entry profile of the EDEM diploma course, which requires teachers who are willing to know and use in their didactic sequences the new trends in Mathematics Education, contemplated in the proposal of fundamental learning rights.

3.3 Design of the Diploma Program

The diploma course was designed for blended learning that participants could follow through the virtual campus of the University of Panama. The course was designed to have certain characteristics of a MOOC (Massive Open Online Course) combined with face-to-face activity. At the beginning of the course, each participant received a physical course folder and an account on the web platform to access the virtual campus.
The use of this platform had an impact on the development of the participants' digital competence, which was also evidenced in their interaction through forums and internal mail (Tangarife, 2018). On the other hand, the entry profile to the diploma course also demanded the mastery of the basic computer tool that would allow participants to be trained with an educational and distance model in online virtual environments. It should be noted that the teachers who did not have this digital competence received specific training beforehand, where they were explained the use of the virtual classroom.
One of the advantages of using the virtual platform is that it makes it possible to demonstrate the use made of it by participants in the EDEM. Figure 2, for example, shows some aspects of the use of the platform by educational region:
Figure 2: Frequency distribution of the use of the platform by Educational Region.



	Coclé	Azuero	Panamá Oeste	Panamá Centro	Veraguas	Colón
	24	27	25	24	27	23
Interactions	24619	27741	25563	27304	23427	18722
Coverage rate	89%	98%	84%	80%	84%	83%
Estimated platform usage time	02:30	03:00	03:10	03:15	02:45	02:35
Number of e-mails sent	48	63	59	55	58	45
Estimated starting time	13:30	17:35	19:25	10:32	11:00	14:20
Estimated completion time	15:45	20:43	22:37	12:45	12:39	16:36

The face-to-face sessions were held at the centers for innovation, technological development and entrepreneurship (CIDETE), at the end of each module the participating teachers attended class where they developed work from a computer, in their own time and space (3 hours per day on average).

From a didactic perspective, the diploma course was designed to favor the competences described in section 3.1, especially the development of the competence of didactic analysis and intervention, especially the development of the subcompetence of didactic suitability assessment, based on the learning and use of the didactic suitability criteria (CID).

The following steps are followed for learning the CIDs: 1) in modules 1-3 some indicators and components are worked on in isolation, 2) in the last final module (4) a narrative of a future teacher about her observation of another teacher's class is presented to them and they are asked, among other questions, to propose an improvement in the implementation of the observed task. 3) It is explained that this assignment asks them to improve the tasks and the question is posed: what criteria should be taken into account to improve a sequence of tasks? 4) A video is shown where this question is reflected upon and the CIDs are introduced for the first time, making them observe that some of them have been taken into account when they have suggested how to improve the task. 5) It is explained that the notion of didactic suitability is composed of six partial didactic suitability criteria, each, in turn, broken down into components and indicators, whose function is to point out aspects to improve in the teacher's practice and that some of these components and indicators have already appeared in the previous modules. For example, in section 1.1 of module one of the Diploma, we reflected on the importance of teaching mathematics from real contexts and close to the student and we make them observe that this reflection suggests the component <<interests and needs>> and the indicators <<Selection of tasks of interest to students>> and <<Proposition of situations that allow us to assess the usefulness of mathematics in everyday and professional life>> of emotional appropriateness. 6) The complete list of criteria, components and indicators are presented and some of them are related to reflections made in the previous modules. Finally, teachers have to design a sequence of tasks, implement them and assess using the CIs (stages d and e of the training cycle of the secondary master's degree).

3.4 Use of the Didactic Suitability Criteria Construct as a Guideline to Guide Practice.

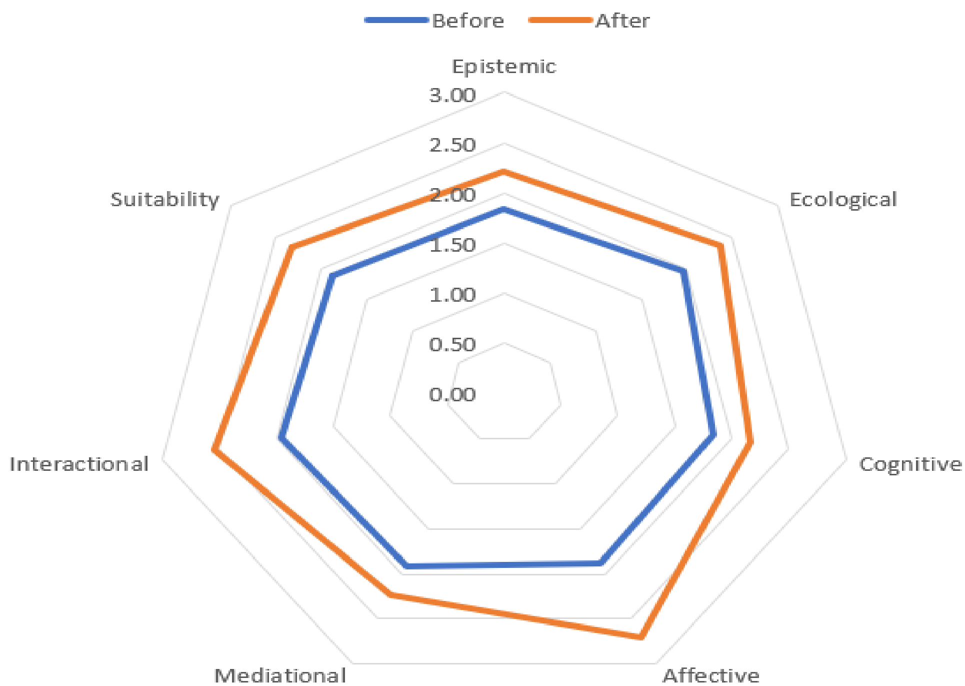
In section 2.2 it was explained that the Didactic Suitability Criteria construct is made operational through a guideline in which the six criteria are broken down into components and indicators (Breda et al, 2017). Based on this guideline, an instrument was designed and validated (see Table 3 in Annex 1) to infer the criteria that the participants had in mind before studying the EDEM diploma course, when they had not received training on the use of the criteria as a guideline to organize their reflection, and after having studied it (since in the diploma course they were trained on the use of the CIDs, especially in the last module that dealt with reflection on practice. In other words, this instrument was applied right at the beginning and end of the diploma course; in particular, the teachers were given an instrument with only the items in the third column of the questionnaire in Table 3 of Annex 1. Each item is a statement that implies considering a criterion and each participant must indicate the degree to which he/she agrees or disagrees with its content.

The validation was first performed by three experts in the area of Mathematics Education, which implied some changes with respect to the first questionnaire developed, in particular the incorporation of some negative items to try to prevent the respondent from leaning towards a certain option; then it was applied to a group of future teachers of the Faculty of Education of the University of Panama, and finally, their responses were analyzed with SPSS 20 software and the reliability of Cronbach's Alpha was 0.935.

For each item, the teacher had to select a value on a likert scale from 0 to 4 that reported the importance of taking the item's criterion into account in the planning and design of his or her task sequence. The SPSS program was used to calculate the means of the responses of all participants to each item, then of each component and, finally, the mean of all the components associated with each criterion of didactic suitability, thus obtaining the blue hexagon in the first application of the questionnaire and the orange hexagon in the second application.

From the comparative analysis of the criteria that teachers took into account before starting the diploma course and those that they stated they considered at the end of the course (Figure 3), it can be observed that before starting the diploma course, teachers implemented didactic units in which the enhancement of motivation, the use of manipulative and technological resources and a unidirectional interaction (blue hexagon) were considered very little. At the end of the diploma course, their didactic units incorporated motivating tasks, in which manipulative and technological resources were used and where interaction was no longer centered on the teacher but among the students.

Figure 3: Representation of the criteria that guide the teacher's practice before and after the diploma course.



4. Conclusiones

This article explains the design guidelines of the EDEM diploma program and some results of its second implementation in a semi-virtual format. It should be noted that the design was based, on the one hand, on a theoretical model on the knowledge and competencies of the mathematics teacher and, on the other hand, on the competencies needed by primary school teachers in Panama according to MEDDUCA and the universities that train teachers, in order to achieve the fit of both perspectives. The conclusion is that the competencies that the training program analyzed seeks to develop are consistent with the competencies currently needed by primary school teachers in Panama. That is, it develops the generic and specific competencies of the graduate profiles of two of the main institutions of higher education that offer teacher training in Panama, and also the

teacher competencies established by MEDUCA in its guiding documents; in addition to developing the competencies proposed by the UNDP.
Among the main results, it is worth highlighting, first, the training program itself, that is, the platform and the set of tasks designed, which can be used for the training of future promotions of teachers (as has been the case). As a second important result, it is worth noting the change in the criteria that guide the teacher's practice at the end of the diploma course in relation to the criteria that the teacher used at the beginning, in particular the greater relevance gained by the criteria related to affective and emotional aspects and those related to interaction management. These results allow us to conclude that the implementation of the EDEM diploma course has had an impact on the way the participating teachers teach their classes. Furthermore, in the last edition of this diploma course, it has been possible to verify that this impact has been transferred to the learning results of the students.
In addition to the above, it is necessary to point out that the diploma course allowed the participating primary school teacher to obtain a certification from the University of Panama and the Ministry of Education (Aval 1097 and 038/PC/2018, respectively) for having received training that, among other aspects, allowed them to incorporate new technologies in their teaching practice, as well as having a vision on the teaching of mathematics that incorporates the new trends on the teaching of this subject, which, in particular, allows the teacher a change on how to teach mathematics and that, in the future, may allow him to teach it within the framework of interdisciplinary proposals (STEM, etc.).

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ANNEX 1 **Table 3**. Questionnaire to infer criteria used by teachers

Epistemic Suitability	Errors	I use correct mathematical notation and language.
		When I make a mathematical error (by mistake) I correct it quickly.
		I am aware of the mistakes that my students make and I try not to validate them.
		I try not to make mistakes in definition, procedure, argumentation, representation and problem posing in my class.
		When I make a mathematical error in the classroom, usually, for various reasons, I do not correct it
	Ambiguities	Students may be ambiguous with my explanations.
		I teach more illustrative mathematics (using metaphors, analogies, gestures or various didactic materials), but I also do not fail to incorporate the correct mathematical definitions, representations, procedures, etc.
		I seek to teach in a meaningful way (using metaphors, analogies, gestures or diverse didactic materials) and, therefore, I do not delve into definitions and correct or appropriate mathematical language.
		I use accurate and appropriate vocabulary in my written and oral developments.
	Richness of processes	I develop alternate ways of solving problems presented in class.
		I do mathematical modeling in class.
		The tasks I set in my classes are mechanistic and based on repetition.
		The tasks I pose encourage experimentation, argumentation, conjecturing, etc.
		I encourage the development of mathematical thinking through problem solving.
	Representativeness of the	You know with little depth the contents you are going to teach.
		I work with different meanings of the same mathematical object and the connection between them considering the educational stage being taught.

	complexity of the object of mathematical teaching	I work with a variety of tasks that explore different meanings, representations, languages, procedures, etc. relevant to the student.
		Working with one mathematical meaning of an object and the problems related to that meaning, not exploring other meanings.
Ecological Suitability	Adaptation to the curriculum	I decide what contents and how to approach them, taking into account the curricular guidelines.
		The contents are in dissonance with those stated in the curriculum.
		You identify the skills that your students should develop in a topic or unit.
		Fully attends to all the standards, processes and concepts indicated by the Curricular Framework in relation to the topic taught.
	Intra- and interdisciplinary connections	Connect the concepts of their specialty with other disciplines and their applications to reality, in an appropriate context for students.
		Integrate the methodology with innovative and teaching strategies in my classes, aligned to contents of other disciplines.
		In the proposed tasks, it is not considered to relate mathematics with other subjects, once mathematics is superior to the others.
		I present several examples in the class that allow the student to relate the subject in a clear and direct way with their experience.
	Socio-occupational usefulness	The instructional techniques used such as: cooperative learning, problem solving, collaborative tasks, among others, allow to see the degree of labor insertion of the subject to the students.
		Integrate the methodology with innovative and teaching strategies in my classes, aligned to the environment of my school.
		Integrate other members of the community in socio-labor issues that help explain the mathematical phenomena of everyday life.
		You make your students aware of the relationship of the subject you teach to their future career curriculum.
	Didactic innovation	You keep a reflective journal about your teaching practice and use it as a tool for self-evaluation and self-improvement.
		I plan each school year using previously done task designs without modifying them.
		Continuously uses new digital resources of the area allowing the student to become familiar with the terminology of mathematics.
		Attends various workshops and courses that instruct him on how to optimize the teaching-learning process to apply it later.
Cognitive Suitability	Prior knowledge	With each new topic you check which concepts are necessarily relevant to develop the new class.
		I explored students' existing knowledge of a topic in order to relate it to a new topic.
		Perform enough teaching and learning activities to promote understanding of a multi-session mathematical phenomenon.
		Your students' prior knowledge is limiting in addressing new knowledge.
	Curricular adaptation	The plan includes relevant tasks to meet learning expectations and demonstrates an effective strategy for providing feedback to students on those tasks.

	to individual differences	Demonstrates sensitivity to the needs of all students with special attention to exceptional and special education students and encourages that sensitivity on the part of the whole group.
		Promotes equitable participation of all students.
		Uses multiple teaching strategies to address diverse learning styles.
	Learning	Accommodates planning to bring all students to the desired level of understanding of the concept or competence with the process.
		Integrates topics with other areas of knowledge in her classes.
		At the end of each class synthesizes the most important points that were reviewed.
		I established mechanisms to ensure that the students' products and performances comply with the development of the skills set forth by the curriculum.
	High cognitive demand	Assignments challenge students to adapt what they have learned and integrate concepts to do the task.
		The examples, diagrams or graphs used allow the students to develop their understanding of the concepts.
		Promote student creativity and originality in the teaching-learning process.
		You encourage your students to develop ideas and/or projects.
Affective Suitability	Interests and needs	The activities or assignments in the textbooks used are of interest to most of the students.
		I use information technologies such as the Internet, videoconferencing to interest my students.
		I take into account the student's opinion in making decisions related to the subject.
		I adapt to the needs of the students to develop the class.
	Attitudes	I encourage the perseverance of my students to solve a proposed task.
		Your objectivity and fair treatment of students is always evident to them and demands fair treatment among them in your class.
		Students are responsible for the tasks assigned to them.
		You consider that the environment you promote in class is adequate for students to achieve learning.
	Emotions	You take advantage of opportunities to foster an appreciation for the beauty and aesthetic value of the subject matter.
		In my classes, I try to show the importance of mathematics through the difficulty that this discipline presents.
		Is dynamic in his/her work as a teacher of mathematics, transmitting his/her enthusiasm to the students.
		Pupils are always accessible to my classes.
Mediation al Suitability	Material resources	Incorporates the use of manipulative materials in the classroom.
		Encourage the use of additional resources in addition to those used in class.
		I construct materials to introduce mathematical concepts.
		I use adequate additional didactic materials to help the student understand the concepts of the class.

	Number of students, timetable and classroom conditions	You adapt your planning to the conditions of the classroom and the number of students and the assigned timetable.
		You adhere to the evaluation systems and criteria specified in the program.
		You find space in your school outside of class time when you need quiet time to work on your own.
		Organize the sequence of activities according to the course schedule.
	Time	During your implementation, eliminate tasks due to lack of time.
		Dedicate the necessary time to solve the students' difficulties.
		Any task that exceeds the stipulated time in a class will be redistributed.
		Invest the time in the less important contents of the subject.
Interactional Suitability	Teacher-student interaction	Shows confidence and assurance in managing interactions, even when it is necessary to review or explore an unplanned topic, or deal with unexpected questions.
		At all times reacts professionally and takes advantage of questions, answers, comments, and even unexpected behavior, to broaden students' understanding of the subject matter.
		You do not ask them about your students' academic problems.
		You make appropriate and effective use of diverse questions that promote classroom discussion to promote understanding of fundamental concepts and processes.
	Interaction between students	Take advantage of a relevant student participation to share it with the group and involve others.
		Effectively use cooperative learning with my students.
		Implements strategies that contribute to improve technological, investigative and educational skills in mathematics among students.
		Promote collaborative participation in some of your students' work.
	Autonomy	Design assignments in which students always play a passive role.
		I try to teach students that good decision making depends on identifying the elements of the problem, so that they are independent in solving a problem.
		Respect the turn in interventions and listen to the speaker, be assertive at that moment.
		Encourages students to take some of the responsibility for the class.
	Formative Assessment	Uses a variety of evaluation techniques in a diagnostic manner to demonstrate the achievement of objectives.
		Does not perform formative activities to promote understanding of a mathematical phenomenon in several sessions.
		Evaluates learning at various times during the course.
		Corrects the students' formative work promptly by discussing it with them and tries to solve their difficulties.