Kurdish Studies
December 2022
Volume: 10, No: 02, pp 1384-1394
ISSN: 2051-4883 (Print) | ISSN 2051-4891 (Online)
www.KurdishStudies.net

DOI: 10.67145/ks.v10i2.4129

A Study Of Fuzzy Data Analysis And It's Mathematical Models

Neeta Verma[1], Dr. Vinita Dewangan[2], Dr. A. J. Khan[3]
[1]Research Scholar, MATS University, Raipur, verma.nvmili@gmail.com.
[2]Asso. Professor Mathematics, MATS University, Raipur, drvinitad@matsuniversity.ac.in
[3]Professor Mathematics, MATS University, Raipur, khanaj@matsuniversity.ac.in

Abstract
This study analyzes fuzzy data and its mathematical models, highlighting their vital role in managing uncertainty across various domains. Employing a quantitative research design, the research assesses key theories and methodologies related to fuzzy data, evaluating the effectiveness of models such as fuzzy logic systems, clustering algorithms, and regression models. While these models provide valuable insights, they also face limitations related to data quality, model scope, and computational demands. The findings emphasize the need for careful model selection and validation for real-world applicability and stress the importance of exploring advanced mathematical approaches alongside ethical considerations in real-world data usage. This research enhances the understanding of fuzzy data analysis, paving the way for future studies aimed at improving model robustness and expanding applications in diverse fields, ultimately informing decision-making processes in uncertain environments.

Keywords: Fuzzy Data Analysis, Mathematical Models, Uncertainty Management, Data Quality, Model Validation

1. Introduction

In today's data-driven landscape, effectively managing uncertainty is essential across various fields, including finance and healthcare. This study explores fuzzy data analysis, which offers a robust framework for addressing inherent ambiguities in real-world data. Utilizing a quantitative research design, the research examines key theories and methodologies related to fuzzy data, evaluating mathematical models such as fuzzy logic systems, clustering algorithms, and regression models. While these models provide valuable insights, challenges regarding data quality, model scope, and computational demands remain. The study underscores the necessity of careful model selection and validation to ensure practical applicability and advocates for exploring advanced mathematical approaches and ethical considerations in data usage. By deepening our understanding of fuzzy data analysis, this research lays the groundwork for future studies focused on improving model robustness and expanding applications across diverse domains.

2.Mathematical Models in Decision-Making

Mathematical models are essential in decision-making processes where multiple, often conflicting, criteria are involved. This aspect is particularly beneficial since most real-world problems are influenced by various factors rather than a single criterion. The primary objective of mathematical methods is to prioritize specific alternatives in scenarios involving multiple decision-makers and criteria across different time periods.While there are various ways to classify mathematical methods, a comprehensive classification is often avoided due to similarities in their underlying models.

Instead, frequently used methods include:

- Points Method
- ELECTRE Method
- PROMETHEE Method
- TOPSIS Method
- Analytic Hierarchical Process (AHP)
- Fuzzy Decision Making
- ANFIS Models (Adaptive Neuro-Fuzzy Inference System)
- Neural Network-Based Models
- Fuzzification Models of Existing Methods

The selection of evaluation methods is influenced by several factors, including:

- The importance and character of the decision to be made.
- The context in which the decision will be implemented.
- The nature of the decision prompting the evaluation.
- The funding mechanisms for implementing new solutions.

In responsible decision-making, specialized analysis and indirect optimization methods are often employed, particularly soft optimization techniques that address multiple objectives—some to be maximized and others minimized. These methods model priority conflicts among decision-makers, aiming to find solutions that approach an ideal point or represent the best compromise. Decision-making typically involves evaluating potential solutions, where single-criterion optimization identifies an optimal solution. However, when multiple criteria are involved, the focus shifts to identifying the best possible solution, as grouping criteria can limit analysis and accuracy. Mathematical models address this complexity through nuanced approaches rather than relying solely on total scalarization. The decision maker or analyst controls the target function scalarization by comparing criteria and assigning ranks based on preferences, creating a matrix of alternatives and criteria for establishing weighting scores. These scores may be used individually or collectively, guiding resource distribution or simply ranking alternatives. Analysts encounter complex environments with conflicting criteria, requiring flexible tools beyond rigid mathematical techniques. Soft optimization methods like promethee, electre, ahp, topsis, and cp utilize heuristic parameters and value scales. Recent developments have integrated standard and fuzzy versions of these models to incorporate human subjectivity and expert knowledge. New fuzzy logic and neuro-fuzzy models have emerged, leveraging military officers' experiential knowledge to create automated decision-making strategies. Fuzzy sets help quantify imprecise information, enhancing fuzzy reasoning in uncertain decision-making

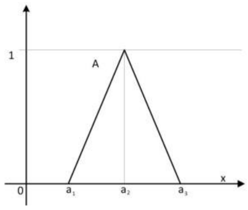

Figure 1 Triangular fuzzy number

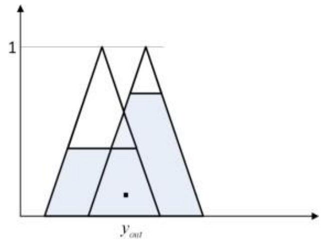

Figure 2 Defuzzification

FM'WOT Model: When designing an organizational structure, various decisions must be made. It is important to note that the subjective evaluation of certain parameters can vary significantly between decision-makers. Fuzzy set theory provides a useful approach for quantifying these parameters.

3.Fuzzy Sets: Fuzzy set theory defines a fuzzy set A as a collection of ordered pairs. This framework allows for the representation of uncertainty and subjective judgments, making it particularly valuable in decision-making contexts. The application of mathematical models within this framework enhances the ability to analyze and interpret complex scenarios effectively:

$$A = \{(x, \mu_A(x)) | x \in X, 0 \le \mu_A(x) \le 1\},$$

Where □ □x□ is a membership function which shows to what extent x □ X meets the criterions for membership in a set A . For the membership function 0 □ □ A □ x□ □ 1, for every x A □ ,that is □ A : X □ □ 0,1□

In fuzzy theory, the selection of membership functions, including their form and the width of confidence intervals, is typically based on subjective estimates or prior experience. Triangular fuzzy numbers are commonly employed due to their simplicity and effectiveness. A triangular fuzzy number A is represented as A(a1, a2, a3), where:

- a2 is the peak value of the membership function, indicating where the membership is 1.0.
- a1 defines the left endpoint of the confidence interval.
- a3 defines the right endpoint of the confidence interval.

The membership function for the fuzzy number A is defined mathematically to capture these characteristics. This approach facilitates the modeling of uncertainty and enables precise quantification in the analysis of fuzzy data.

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3 \\ 0, & x > a_3 \end{cases}$$

For defuzzification and mapping of the fuzzy number,

A = (a1, a2, a3) Value into a real number, numerous methods are used (Figure 2). Two methods have been used in this parer:

The center of gravity:

defuzzy A = [(a3 – a1) + (a2 – a1)] 3-1 + a1

The total integrate value:

Deffuzzy A = [□ a3 + a2 +(1 - □) a1] 2-1

(with □ , □ € [0, 1] being an optimism index).

Basic Model: In decision-making scenarios, organizations explore various alternatives to determine the most suitable course of action. The fuzzy mathematical model (FMM) outlined below aids in assessing different solutions and identifying optimal options. Additionally, SWOT analysis is utilized to evaluate which factors should be eliminated, retained partially, or fully maintained, thereby informing the design strategies for the organizational structure. By analyzing strengths, weaknesses, opportunities, and threats pertinent to the organization, an effective optimization model is proposed.

The fuzzy mathematical model comprises the following steps:

Step 1: Identify the SWOT sub-factors and determine alternative strategies based on these sub-factors. Assess the significance of the SWOT factors. If the model is applied to evaluate alternatives for previously proposed organizational structures, this step may be omitted. The subsequent steps represent a general case where K is assessed to identify the best alternative from a finite set of alternatives A={a1,a2,…,an} The optimal criteria are formally represented as K={k1,k2,…,km}, where K indicates the total number of criteria considered. The decision-making problem is mathematically represented by the matriF with dimensions K × A. This model allows for the application of fuzzy set theory to evaluate and optimize decision-making processes effectively.

$$F = \begin{array}{c c} & \begin{array}{ccccc} A_1 & \cdots & A_{n-m} & \cdots & A_n \end{array} \\ \begin{array}{c} K_1 \\ \vdots \\ K_k \\ \vdots \\ K_K \end{array} & \begin{bmatrix} f_{11} & \cdots & f_{1n-m} & \cdots & f_n \\ \vdots & & & & \vdots \\ f_{k1} & \cdots & f_{kn-m} & \cdots & f_{kn} \\ \vdots & & & & \vdots \\ f_{K1} & \cdots & f_{Kn-m} & \cdots & f_{Kn} \end{bmatrix} \end{array}$$

Where fki □ I □ 1, A, K = 1, k□ is the linguistic or numerical value of the optimum criterion K ∈ K for alternatives a (a∈ A). If at least one criterion is described by linguistic expression in the description of the optimum criterion, step 2 is taken.

Step 2: Defining the Set of Linguistic Descriptors: Criterion values are represented using a set of linguistic descriptors denoted as S={l1,l2,…,lT}. where T is the total number of linguistic descriptors. These linguistic variables are characterized by triangular fuzzy numbers, represented as A=(a1, a2,a3). This framework allows for the effective representation of uncertainty and vagueness in the decision-making process.

Step 3: Normalisation of the optimisation criterion: For the value fki □ i □ 1,A,k □ 1, k□ to be comparable, it is necessary to normalise them. If the optimisation criteria are given as linguistic values fki □ i □ 1,A, K□ fki □□ fki, ,□ fki□ , normalisation is performed as follows:

i) For the benefit criterion K □ k□ K□ normalisation is performed as follows:

$$(f_{ki})_n = f_{ki} \,/\, f_k^{max} \qquad (1)$$

Where max f kmax is max value of the fuzzy number f ki (k = 1,….,K) , for ,□ fk □ fk□ □ 0

2) For the cost criterion K □ k □ K □ normalisation is performed as follows:

$$(f_{ik})_n = 1 - f_{ki} - f_k^{min} / f_k^{max} \qquad (2)$$

Where min fkmin is minimum value of the fuzzy number fki(K = 1,…..,K), for □ fki □ 0. The normalised value of the criterion for k (k € K) a (a □ A) alternative is described by fuzzy number:

$$(f_{ik})_n = \{(f_{ik})_n \,, \mu_{fik}\} \qquad (3)$$

If the optimality criteria are described in numerical values fki □ i□ 1, A, K □ 1,K□ , normalisation is performed as follows:

$$f_{ki} = \frac{f_{ki}}{\sum_{k=1}^{K} f_{ki}}, \sum_{k=1}^{K} f_{ki} = 1 \qquad (4)$$

Step 4: Evaluation Criteria: The set of optimality criteria is denoted as K={k1, k2,…,kk}, where KKK represents the total number of criteria considered. Each criterion can be further disaggregated into sub-criteria. If jk denotes the total number of sub-criteria under the j-th criterion, then the overall number of criteria can be expressed as:

$$K = \sum_{j=1}^{n} k_j \quad (5)$$

Each criterion must be divided into sub-criteria. In this case, the sum of the sub-criteria for a given criterion equals 1. This is crucial for understanding how judgments are aggregated across two consecutive hierarchical levels, where both criteria and sub-criteria are involved. The aggregation process involves shifting the criteria to the sub-criteria level, after which the original criteria level no longer exists. The relative importance of the optimality criterion k ∈ K changes. The value representing the importance of the optimality criteria is determined by constructing a matrix Wkij, where the elements are linguistic descriptors and numerical values that indicate the significance of criterion k ∈ K relative to criterion k' ∈ K'. Once the W matrix is established, normalization of the weight coefficients is performed.

$$w_{kij} = \frac{\lambda_j w'_{k_j}}{\sum_{j=1}^{k} \lambda_j w'_{k_j}}, \sum_{j=1}^{K} w_k = 1, w_k \text{ ó } [0,1] \qquad (6)$$

Here, λj represents the decision maker's preference for attribute i. The process of designing the organizational structure is typically handled by multiple experts, or decision-makers. In such cases, the evaluation of the optimality criteria from all group members should first be collected for synthesis before proceeding to step 5. In other instances, step 6 is taken.

Step 5: Evaluating Criteria in Group Decision-Making: In group decision-making, synthesis can occur with either complete or incomplete information. When group synthesis involves complete information, and all members of the group G (where e=1,2,…,n) are treated equally in the decision-making process, and the evaluations of the optimality criteria for the given hierarchy have been completed, there are two methods for prioritizing alternatives in relation to the goal. One approach is to aggregate all the priority criterion vectors obtained from each decision-maker using the following equation.

$$w_i^G = \sum_{e=1}^{n} \alpha_e w_i(e) \qquad (7)$$

w(ei) represents the weight value assigned to the nth member of group G (where e=1,2,…,n) for the alternative Ai. The term αe denotes the weight value (significance) of the nth group member, while Gwi represents the overall priority of alternative Ai. The individual weights of the group members, αe, are first normalized using an additive approach. However, a limitation of this procedure is that it cannot be applied in cases of group synthesis with incomplete information, as certain members of the group may lack composite vectors. An alternative method is to aggregate all individual preference assessments across all hierarchical levels immediately.

$$w'_i = \left[\prod_{j=1}^{n} \lambda_j a_{ij}\right]^{1/n} \rightarrow w_{kij} = \frac{\lambda_j w'_{k_j}}{\sum_{j=1}^{k} \lambda_j w'_{k_j}} = \left[\prod_{j=1}^{n} \lambda_j a_{ij}\right]^{1/n} \left\{\sum_{i=1}^{n} \left[\left[\prod_{j=1}^{n} \lambda_j a_{ij}\right]^{1/n}\right]\right\}^{-1} \sum_{j=1}^{K} w_k = 1, w_k \text{ó}[0,1] \qquad (8)$$

Here, λj represents the decision maker's preference for attribute i. In cases of group synthesis with incomplete information, the micro-aggregation of the (i,j) position in the given matrix is performed using the geometric mean of the assessments from those group members who expressed a preference for Ei in relation to the element Ej. A key requirement in this scenario is that at least one decision maker must provide a declaration on the value of aij. This modifies the previous expression

$$w_i^G = \left[\prod_{l \in L}^{n} \lambda_j a_{ij}(l)\right]^{1/M} \rightarrow w_{kij} = \frac{\lambda_j w_{k_j}^G}{\sum_{j=1}^{k} \lambda_j w_{k_j}^G} = \left[\prod_{j=1}^{n} \lambda_j a_{ij}\right]^{1/n} \left\{\sum_{i=1}^{n} \left[\left[\prod_{j=1}^{n} \lambda_j a_{ij}\right]^{1/M}\right]\right\}^{-1} \sum_{j=1}^{K} w_k = 1, w_k \text{ó}[0,1] \qquad (9)$$

Here, l represents the set of group members who have assessed the pair of elements (Ei,Ej), and M denotes the number of those members.

Evaluating Alternatives: After determining the weight coefficients for all assessed criteria, amatrix $F\ [F_{ij}^{e}]_{fw}$ is constructed, where the elements cij of the matrix are derived using the following expression:

$$c_{ij} = \frac{fki}{\sum_{i=1}^{J} fki} Wk_{ij} \qquad (10)$$

Here, fki represents the value of the criterion function for alternative AiA_iAi (where i=1,…i= 1, \ldotsi=1,…) and criterion k (where k ∈ K, while wk denotes the weight coefficient for criterion k (where k ∈ K. , additive synthesis is assumed, and the final performance weights of the alternatives concerning the overall goal are calculated by summing the elements in the
rows of the performance matrix $F[F_{ij}^{e}]_{fw}$:

$$ei = \sum_{j=1}^{k} Cij,\ Wj \qquad (11)$$

The value of the criterion functions Vi for each assessed alternative is derived from the matrix F using the following expression:

$$Vi = \sum_{j=1}^{k} cj,\ (k \in K) \qquad (12)$$

To rank the alternatives, it is essential to prioritize the aggregated assessments. Since each Vi is represented as a triangular fuzzy number, a method for ranking triangular fuzzy numbers must be applied. Several methods are available for this purpose, including the center of gravity method, the dominance measure method, the α\alphaα-cut with interval synthesis method, and the total integral value method. Among these, the total integral value method (Liou and Wang, 1992) is considered an effective choice for accomplishing the task efficiently and has therefore been proposed within this methodology. For the triangular fuzzy number A=(a1,a2,a3), the total integral value is defined as follows:

$$I_T^{\lambda} = (A) = [\lambda a_3 + a_2 + (1-\lambda)]2^{-1}, \lambda \in [0,1] \qquad (13)$$

In Equation 13, λ represents an optimism index that reflects the decision maker's attitude toward risk. A higher value of λ signifies a greater degree of optimism. In practical applications, the values 0, 0.5, and 1 are typically used to represent pessimistic, moderate, and optimistic perspectives, respectively. fVi = max (fvi), i + 1,…,A. For the given fuzzy numbers A and B, it is stated that if Iλ (A) < Iλ (B). then A < B; If Iλ (A) > Iλ (B). then A = B; and if Iλ (A) > Iλ (B), then A > B . The final ranking of alternatives involves selecting a specific level of optimism (λ) from the decision-maker. Subsequently, Equation 13 is applied to the fuzzy numbers described in Equation 12, allowing for the ranking of alternatives based on the derived values for Iλ (F), i =1,…, N. The optimal alternative from this set is denoted as.

4.Designing the Management Organizational Structure of Traffic Support Using the FM'WOT: Model

The design of a military management system significantly influences the creation, adaptation, functionality, and quality of operations. No military organizational system can operate independently from its management subsystem, which issues commands to guide desired behaviors, even if actual behavior deviates. To effectively serve numerous traffic and transport service users while meeting military requirements, an organizational structure must be established. Within the General Staff and the Logistics Department, traffic management acts as the highest professional traffic authority of the Serbian Armed Forces, overseeing conditions, development, organization, monitoring, training, normative regulations, and traffic control support. The events of the 1990s prompted a reassessment of the organizational structure of transport support governing bodies, highlighting the need for a deeper examination of traffic management frameworks.

Illustrative Application of SWOT Analysis:

A SWOT analysis was conducted to evaluate external influences, opportunities, and threats impacting traffic support management. This analysis aids in managing the organization, including its structural components. Four alternatives for the organizational structure of traffic support governing bodies were identified:

- Alternative 1 (A1): Reflects the current organization based on normative regulations and military decision-makers' experiences, comprising the Department of Traffic Operations and the Department of Transport.
- Alternative 2 (A2): Aligns with NATO standards for traffic support governing bodies.
- Alternative 3 (A3): Organized by specific processes, with units defined for each process and specialists integrated into groups. Traffic management oversees these processes.
- Alternative 4 (A4): Employs a logistic approach and functional organizational principles to define the authority of traffic support governing bodies, illustrating internal labor division, differentiation, specialization, and command control.

Assessment, Synthesis, and Ranking: The initial steps in applying the fuzzy mathematical model involve defining a set of linguistic descriptors. Linguistic variables are represented by a collection of linguistic descriptors: S={l1,l2,…,li}, where I ∈ H = {0,1,…,T}, In this context, T denotes the total number of linguistic descriptors, which is set at T=9T = 9T=9. The linguistic descriptors are as follows: Unessential (U), Very Low (VL), Fairly Low (FL), Low (L), Medium (M), High (H), Medium High (MH), Very High (VH), Perfect (P). These linguistic descriptors correspond to specific values, as illustrated in Figure 3. This framework serves as a foundational component in the study of fuzzy data analysis and its mathematical models,

$$\mu lM = \begin{Bmatrix} 0, & 0 < x \\ \frac{(0.125-x)}{0.125} & 0. \le x \le 0.125 \end{Bmatrix} \quad (14) \qquad \mu lM = \begin{Bmatrix} 0.125, & 0 \le x \le 0.125 \\ \frac{0.250-x}{0.125} & 0.125 \le x \le 0.25 \end{Bmatrix} \quad (15)$$

$$\mu lM = \begin{Bmatrix} \frac{x-0.125}{0.125}, & 0.125 \le x \le 0.250 \\ \frac{0.375-x}{0.125} & 0.250 \le x \le 0.375 \end{Bmatrix} \quad (16) \qquad \mu lM = \begin{Bmatrix} \frac{x-0.50}{0.125}, & 0.50 \le x \le 0.625 \\ \frac{0.75-x}{0.125} & 0.625 \le x \le 0.75 \end{Bmatrix} \quad (17)$$

$$\mu lM = \begin{Bmatrix} \frac{x-0.375}{0.125}, & 0.375 \le x \le 0.50 \\ \frac{0.625-x}{0.125} & 0.50 \le x \le 0.625 \end{Bmatrix} \quad (18) \qquad \mu_{lM} = \begin{Bmatrix} \frac{x-0.50}{0.125}, & 0.50 \le x \le 0.625 \\ \frac{0.75-x}{0.125} & 0.625 \le x \le 0.75 \end{Bmatrix} \quad (19)$$

$$\mu_{lMH} = \begin{Bmatrix} \frac{x-0.625}{0.125}, & 0.625 \le x \le 0.75 \\ \frac{0.875-x}{0.125} & 0.75 \le x \le 0.875 \end{Bmatrix} \quad (20) \qquad \mu_{lvH} = \begin{Bmatrix} \frac{x-.075}{0.125}, & 0.75 \le x \le 0.875 \\ \frac{(1-x)}{0.125} & 0.875 \le x \le 1 \end{Bmatrix} \quad (21)$$

$$\mu_{lp} = \begin{Bmatrix} \frac{x-0.875}{0.125}, & 0.875 \le x < 1 \\ 1, & x \ge 1 \end{Bmatrix} \quad (22)$$

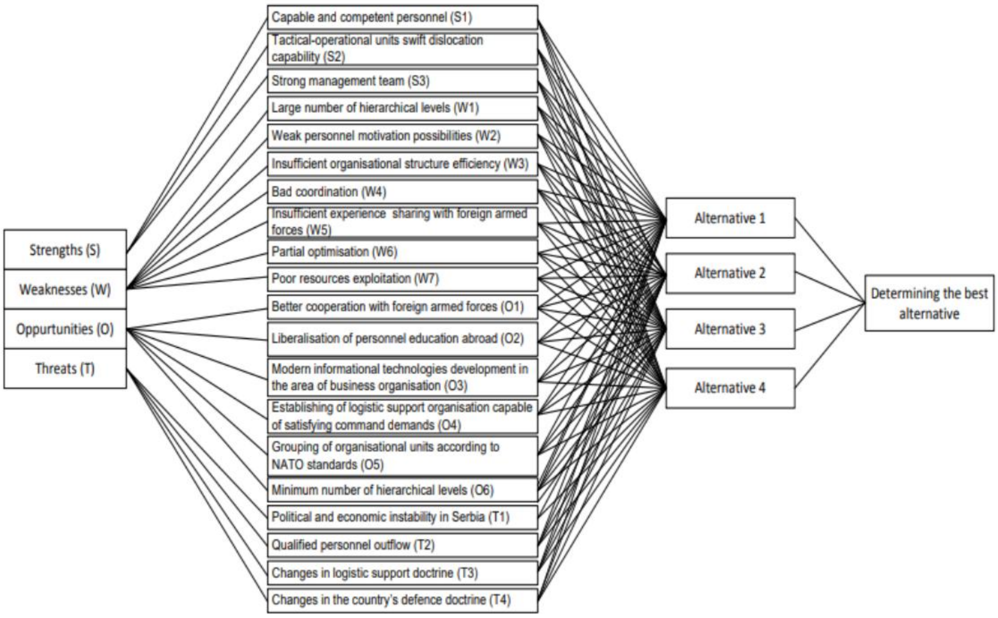

Figure 3 FMM model for SWOT.

To determine the relative importance of the SWOT evaluation criteria, a pair-wise comparison was conducted using fuzzy logic in relation to the goal. Table 1 presents the linguistic evaluations of criteria for each of the given alternatives based on the observed optimality criteria. Linguistic preferences between criteria were used to construct the judgment matrix WWW, as outlined in Step 4. After the decision-maker's assessment of the criteria, normalization of the optimality criteria was carried out using Equations 1 and 2. The weighting vector w~kij\tilde{w}_{kij}w~kij of the criteria matrix W was calculated using Equation 6. Each element of this vector represents the sum of the values in the corresponding row of matrix W, divided by the sum of all elements in the matrix.

Table 1 Optimality criterion level of influence on the observed alternatives.

Criteria and Sub-Criteria	Linguistic Criteria	Benefit-Cost Criteria	A1	A2	A3	A4	Min	Max
Strengths								
Capable and competent personnel	M	MH	H	MH	H	H	MH	H
Tactical-operational units swift dislocation capability	M	VH	H	VH	H	MH	MH	VH
Strong management team	M	VH	H	VH	H	MH	MH	VH
Weaknesses								
Number of hierarchical levels	H	M	H	M	M	M	M	H
Personnel motivation possibilities	L	VH	L	VH	H	H	L	VH
Organisational structure efficiency	L	VH	L	VH	M	MH	L	VH
Coordination	VL	VH	VL	VH	M	VH	VL	VH

Sharing experience with foreign armed forces	VL	VH	VL	VH	M	P	P	VH
Partial optimisation	VH	VL	VH	VL	L	FL	FL	VH
Resources exploitation	M	VH	M	VH	H	VH	M	VH
Opportunities								
Cooperation with foreign armed forces	L	VH	L	VH	MH	VH	L	VH
Liberalisation of personnel education abroad	M	VH	M	VH	MH	VH	M	VH
Modern informational technologies introduction in the area of business organisation	L	VH	L	VH	M	VH	L	VH
Establishing of logistic support organisation capable of satisfying command demands	VL	VH	VL	VH	M	H	VL	VH
Grouping of organisational units according to NATO standards	L	H	L	H	M	VH	L	VH
Participation in logistic support of the NATO forces	VL	MH	VL	MH	M	VH	VL	VH
Threats								
Political and economic instability in Serbia	VH	M	VH	M	VH	H	M	VH
Qualified personnel outflow	VH	MH	VH	MH	VH	MH	MH	VH
Changes in logistic support doctrine	VH	M	VH	M	H	H	M	VH
Changes in the country's defence doctrine	VH	M	VH	M	H	H	M	VH

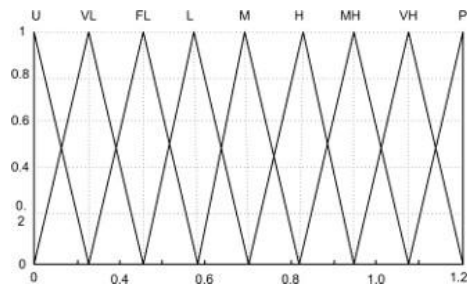

Figure 4 Linguistic descriptors.

The alternatives, according to the observed optimality criteria, are provided. Linguistically expressed preferences among the criteria have been used to construct a judgment matrix W, as outlined in step 4. Following the decision-maker's evaluation of the criteria, the normalization of optimality criteria was carried out using Equations (1) and (2). The weighting vector wkij of the criteria matrix W was determined by applying Equation (6). Each entry in this vector corresponds to the sum of the elements in the associated row of matrix W, which is then divided by the sum of all elements in the matrix.

$$W_{SWOT}=\begin{bmatrix} w_s \\ w_w \\ w_o \\ w_t \end{bmatrix}=\begin{bmatrix} (0.32,0.30,0.27) \\ (0.40,0.36,0.32) \\ (0.16,0.18,0.21) \\ (0.12,0.15,0.20) \end{bmatrix}=\begin{bmatrix} 0.33 \\ 0.29 \\ 0.20 \\ 0.17 \end{bmatrix}$$

Table 2 The priorities of the SWOT factors and groups.

SWOT	Local Priority	SWOT Factors	Global Priorities
Strengths	0.33	Capable and competent personnel	0.39
		Tactical-operational units swift dislocation capability	0.26
		Strong management team	0.35
Weaknesses	0.29	Number of hierarchical levels	0.12
		Personnel motivation possibilities	0.13
		Organisational structure efficiency	0.14
		Coordination	0.17
		Sharing experience with foreign armed forces	0.1
		Partial optimisation	0.16
		Resources exploitation	0.18
Opportunities	0.2	Cooperation with foreign armed forces	0.15
		Liberalisation of personnel education abroad	0.18
		Modern informational technologies introduction in the area of business organisation	0.09
		Establishing of logistic support organisation capable of satisfying command demands	0.2
		Grouping of organisational units according to NATO standards	0.12
		Participation in logistic support of the NATO forces	0.12
Threats	0.17	Political and economic instability in Serbia	0.35
		Qualified personnel outflow	0.07
		Changes in logistic support doctrine	0.3
		Changes in the country's defence doctrine	0.27

In the next step, judgment matrices for the sub-criteria associated with their respective criteria were derived. The weighting vectors for the related sub-criteria were computed as defined by Equation (6) (Table 2). After determining the values of the weighting coefficients for each of the observed criteria, the matrix F =[Wij·cij] was constructed (Table 3). The elements of this matrix cij were calculated using Equation (10).

$$F = \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array}$$
$$F = \begin{bmatrix} (0.30,0.32,0.36) & (0.36,0.32,0.30) & (0.16,0.19,0.21) & (0.12,0.16,0.19) \\ (0.20,0.21,0.23) & (0.24,0.26,0.23) & (0.19,0.21,0.23) & (0.30,0.33,0.35) \\ (0.07,0.08,0.11) & (0.29,0.31,0.32) & (0.26,0.27,0.29) & (0.26,0.27,0.29) \\ (0.14,0.14,0.17) & (0.30,0.31,0.32) & (0.21,0.23,0.25) & (0.29,0.31,0.34) \end{bmatrix} \times \begin{bmatrix} 0.33 \\ 0.29 \\ 0.20 \\ 0.17 \end{bmatrix}$$

The assessment of alternatives was carried out using Relations (10) and (12). The final performance weights of the alternatives, with respect to the overall goal, were calculated using Equation (32). For typical values of λ, which reflect the decision-maker's risk attitude, the final ranking of alternatives was determined by applying Equation (13). The normalized values in Table 4 indicate that Alternative 2

$$V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} (0.20,0.21,0.23) \\ (0.28,0.29,0.32) \\ (0.20,0.22,0.24) \\ (0.22,0.26,0.27) \end{bmatrix}$$

ranks the highest, followed by Alternatives 4, 3, and 1, regardless of the decision-maker's optimism level. By applying the center of gravity method to defuzzify the previously given V values, the final weights of the alternatives after normalization were: 0.213 (Alternative 1),
0.297 (Alternative 2), 0.220 (Alternative 3), and 0.253 (Alternative 4). Clearly, the final ranking aligns with the previous results. An organization is not merely a collection of mechanical parts, but rather an "organic whole" with a defined purpose and mission. When designing an organizational structure, it is essential to not only define the objectives and design criteria but also analyze the organization's current state. In uncertain environments, there is inherent uncertainty and imprecision in the criteria used for organizational design. The fuzzy multi-criteria approach developed in this paper provides a way to quantify these uncertainties and select the best alternative from the proposed organizational models.

Table 3 Additive synthesis.

SWOT Factors	Alternative 1	Alternative 2	Alternative 3	Alternative 4	W2	W1
Strengths						
S1	(0.119, 0.113, 0.109)	(0.119, 0.113, 0.109)	(0.078, 0.084, 0.088)	(0.078, 0.084, 0.088)	0.39	0.33
S2	(0.129, 0.103, 0.092)	(0.129, 0.103, 0.092)	(0.000, 0.026, 0.037)	(0.000, 0.026, 0.037)	0.26	
S3	(0.116, 0.108, 0.102)	(0.116, 0.108, 0.102)	(0.077, 0.080, 0.083)	(0.040, 0.053, 0.062)	0.35	
Weaknesses						
W1	(0.027, 0.028, 0.030)	(0.027, 0.028, 0.030)	(0.018, 0.021, 0.024)	(0.046, 0.042, 0.036)	0.12	0.29
W2	(0.030, 0.031, 0.032)	(0.045, 0.042, 0.039)	(0.030, 0.031, 0.032)	(0.030, 0.031, 0.032)	0.13	
W3	(0.024, 0.027, 0.030)	(0.036, 0.036, 0.037)	(0.024, 0.027, 0.030)	(0.060, 0.053, 0.045)	0.14	
W4	(0.016, 0.022, 0.028)	(0.046, 0.045, 0.046)	(0.030, 0.033, 0.037)	(0.076, 0.067, 0.056)	0.17	

W5	(0.028, 0.027, 0.027)	(0.028, 0.027, 0.027)	(0.018, 0.020, 0.022)	(0.028, 0.027, 0.027)	0.1	
W6	(0.040, 0.040, 0.040)	(0.040, 0.040, 0.040)	(0.040, 0.040, 0.040)	(0.040, 0.040, 0.040)	0.16	
W7	(0.025, 0.029, 0.035)	(0.063, 0.058, 0.052)	(0.025, 0.029, 0.035)	(0.063, 0.058, 0.052)	0.18	
Opportunities						
O1	(0.017, 0.021, 0.027)	(0.043, 0.042, 0.040)	(0.043, 0.042, 0.040)	(0.043, 0.042, 0.040)	0.15	0.2
O2	(0.023, 0.028, 0.034)	(0.058, 0.055, 0.050)	(0.058, 0.055, 0.050)	(0.035, 0.037, 0.042)	0.18	
O3	(0.012, 0.014, 0.017)	(0.031, 0.029, 0.026)	(0.031, 0.029, 0.026)	(0.012, 0.014, 0.017)	0.09	
O4	(0.023, 0.028, 0.036)	(0.058, 0.056, 0.054)	(0.058, 0.056, 0.054)	(0.058, 0.056, 0.054)	0.2	
O5	(0.018, 0.021, 0.025)	(0.046, 0.042, 0.037)	(0.009, 0.014, 0.019)	(0.046, 0.042, 0.037)	0.12	
O6	(0.014, 0.017, 0.022)	(0.035, 0.034, 0.032)	(0.035, 0.034, 0.032)	(0.035, 0.034, 0.032)	0.12	
Threats						
T1	(0.050, 0.059, 0.071)	(0.127, 0.118, 0.106)	(0.050, 0.059, 0.071)	(0.127, 0.118, 0.106)	0.35	0.17
T2	(0.009, 0.011, 0.013)	(0.022, 0.021, 0.019)	(0.013, 0.014, 0.016)	(0.022, 0.021, 0.019)	0.07	
T3	(0.076, 0.076, 0.076)	(0.076, 0.076, 0.076)	(0.076, 0.076, 0.076)	(0.076, 0.076, 0.076)	0.3	
T4	(0.069, 0.069, 0.069)	(0.069, 0.069, 0.069)	(0.069, 0.069, 0.069)	(0.069, 0.069, 0.069)	0.27	

Table 4 Final ranking of alternatives.

Decision alternative	Index of optimism			
	λ=0.0 (pessimistic)	λ=0.5 (moderate)	Λ = 1.0 (optimistic)	Final rank

Alternative 1	0.205	0.212	0.220	4
Alternative 2	0.285	0.295	0.305	1
Alternative 3	0.210	0.220	0.230	3
Alternative 4	0.245	0.255	0.265	2

This model facilitates the evaluation of organizational structure options by accommodating multiple optimality criteria, including both benefits and costs, expressed as numerical values or fuzzy linguistic descriptors. It supports group decision-making, addressing both complete and incomplete information. The model enables comparison of criterion function outputs using defuzzification methods, such as the center of gravity and total integral value method. Its application is demonstrated in designing organizational structures for transport support governing bodies, emphasizing the necessity of a planned and methodical approach to organizational design.

5.Conclusion

This study offers a comprehensive examination of fuzzy data analysis and its mathematical models, highlighting their significance in addressing uncertainty across various domains. Using a quantitative research design, the study identifies key theories and methodologies relevant to fuzzy data and demonstrates the effectiveness of models like fuzzy logic systems, clustering algorithms, and regression models. While these models provide valuable insights, they face limitations related to data quality, model scope, and computational demands. The findings emphasize the importance of careful model selection and validation for real-world applicability and highlight the need for ongoing exploration of advanced mathematical approaches and ethical considerations surrounding real-world data use. Overall, this research enhances understanding of fuzzy data analysis, informing better decision-making in uncertain environments.

References

1. Ali, S., et al. (2024). Extension of interaction geometric aggregation operator for material selection using interval-valued intuitionistic fuzzy hypersoft set.
2. Dağıstanlı, H. A. (2024). An interval-valued intuitionistic fuzzy VIKOR approach for R&D project selection in defense industry investment decisions.
3. Lo, H. W. (2024). A novel interval neutrosophic-based group decision-making approach for sustainable development assessment in the computer manufacturing industry.
4. Mahmood, T., et al. (2023). Analysis and application of Aczel-Alsina aggregation operators based on bipolar complex fuzzy information in multiple attribute decision making.
5. Mousavi, S. M., et al. (2021). ELECTRE I-based group decision methodology with risk preferences in an imprecise setting for flexible manufacturing systems.
6. Deng, Q., Gönül, S., Kabak, Y., Gessa, N., Glachs, D., Gigante-Valencia, F., & Thoben, K. D. (2019). An ontology framework for multisided platform interoperability. In K. Popplewell, K. D. Thoben, T. Knothe, & R. Poler (Eds.),
7. Abdelfattah, W. (2019). Data envelopment analysis with neutrosophic inputs and outputs.
8. Li, A., et al. (2019). Complex neural fuzzy system and its application on multi-class prediction—A novel approach using complex fuzzy sets, IIM, and multi-swarm learning.
9. Wu, J., Guo, S., Huang, H., Liu, W., & Xiang, Y. (2018). Information and communications technologies for sustainable development goals: State-of-the-art, needs, and perspectives.
10. Zaurbekov, N., Aidosov, A., Zaurbekova, N., Aidosov, G., Zaurbekova, G., & Zaurbekov, I. (2018). Emission spread from mass and energy exchange in the atmospheric surface layer: Two-dimensional simulation.