

Transition of Flow Past Five Square Obstacles in Tandem Arrangement at Distinct Gap Spacing

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Abstract:

A two-dimensional (2-D) numerical simulation was performed for the flow past five square obstacles of similar size 'D' aligned inline at fixed Reynolds number $Re = 150$ to investigate the transition in flow behavior. The gap spacing ($g_c = s/D$) between the obstacles was varied from $g_c = 0.25$ to 5. The results were obtained in terms of vorticity contour visualization, time-history analysis of drag (C_d) and lift (C_l) forces, and physical parameters. Four different flow modes were identified under the effect of gap spacing ($g_c = s/d$). These are: i) Shear layer flow mode ii) Half-developed irregular vortex shedding flow mode iii) Fully-developed two-row vortex shedding flow mode iv) Two-row single bluff body flow mode for obstacles C1-C5. The values of C_{dmean} , $C_{d rms}$, $C_{l rms}$, and St were calculated for physical parameters. C_{dmean} exhibited a mixed trend with increasing gap spacing for obstacles C3, C4, and C5. The highest value of C_{dmean} was observed at $g_c = 5$ for obstacle C1, which was 1.4594, while the smallest value was attained for obstacle C3 at $g_c = 0.25$. There were also some negative values of C_{dmean} for obstacle C2 at $g_c = 0.25 - 2.25$ and for obstacle C3 at $g_c = 0.25 - 1.25$ due to the effect of pressure, known as thrust. Furthermore, the Strouhal number showed an increasing trend for both small and large gap spacings, i.e., $g_c = 0.25 - 0.75$ and $g_c = 1.5 - 5$. The highest value of the Strouhal number was observed at $g_c = 5$ for obstacle C5, which was 0.1521.

Keywords: Drag and lift coefficients, Steady flow, Force statistics, square obstacles, Gap spacing.

1. Introduction

Fluid Flow around single or multiple objects in different configurations such as tandem, side by side and staggered arrangements have been observed in various filed of engineering, such as chimneys, cooling devices, micro-electro-mechanical systems, construction of buildings and bridges etc. These structures are faced by fluid-induced forces, when they interacted with fluids like air or water. Therefore, it is important to investigate the resulting effects of their interaction, which may be related to structural vibration phenomenon. Many studies are available in literature to investigate the structural vibration phenomena by using more than one objects of similar diameters that may be circular/rectangular or square [Hishikar et al., (2001), Bovand et al., (2016), Salwa et al., (2021), Salwa et al., (2020) and Salwa et al., (2018)]. Zhou et al., (2006) performed an experiment for study the flow structure mechanism and heat transfer in between the region of two tandem circular obstacles by using gap spacing from $g_c = 1.3$ to 6 at $Re = 7000$. They examined four distinct types of flow mods. Sohankar and Etminan (2009) numerically studied the phenomena of forced convection heat transfer in laminar flow past tandem square obstacles at $Re = 1 - 200$. They studied the effects of Reynolds number on flow mechanism and heat transfer at fixed value of gap spacing. Xu and Zhou (2004) experimentally measured the St values for flow past two circular obstacles by taking the range of Reynolds number from 800 to 4.2×10^4 and $g_c = 1$ to 15 for tandem arrangement. They examined critical gap spacing within the range from $g_c = 3.5$ to 5. Liu and Chen (2002) experimentally studied flow past over tandem square obstacle at $Re = 2000 - 16000$ and examined two various types of flow mods by progressively increasing and decreasing the gap spacing from $g_c = 1.5-9$. Sohankar (2011) numerically examined the influence of gap spacing for flow past two tandem square obstacles by taking $g_c = 3 - 12$ at $Re = 130, 150$ and 500. They notified three different types of flow mods. Inoue et al., (2006) carried out numerical study for flow past two tandem square obstacles for the sound generation in a uniform flow at low Mach number with $Re = 150$. Liu (2003) performed numerical study for flow past one-, two-, three- and four tandem circular obstacles of identical diameter and found the critical gap spacing between $g_c = 3.5$ and 4 for Re ranging from 27,000 to 86,000. He found a sharp increase in drag coefficient for middle and downstream obstacles at critical value of g_c . Tatsutani et al., (1993) performed numerical simulations for two tandem square obstacles in a channel for Re ranging between 200 to 1600. They observed distinct flow mods that are dependent on a critical inter-obstacle spacing. An experimental study on the flow characteristics around three circular obstacles is conducted by Igrashi and Suzuki (1984) and divided the flow behavior in three types. (i) without reattachment, (ii) with reattachment and (iii) rolling up. Moreover, they examined that drag force of central obstacle was negative for the spacing values $g_c < 3.53$. Harichandan and Roy (2010) numerically examined the flow over three inline circular obstacles at $Re = 100$ and 200 at different values of g_c and found that the obstacle placed at downstream location was

experienced huge amount of unsteady forces that caused the damaging of the structure. At $Re = 100$ and $g_c = 2$, they observed steady flow mod. An experimental study was conducted by Lam and Lo (1992) for the flow past over four obstacles at different values of gap spacing in the range $g_c = 1.28 - 5.96$ and $Re = 2.1 \times 10^2$ through induced fluorescent visualization. They examined three different flow modes. A (2-D) numerical study is conducted for flow past an inline configurations of square obstacles by Patil and Tiwari (2009) to find out the behavior of flow field at different ranges of g_c and Re . They found vortex shedding suppression at value of g_c greater than 1. The Cd_{mean} of first obstacle contained positive and end obstacle was attained negative values because of increasing the values of gap spacing. A numerical simulation for flow past over four square obstacles through LBM to study the effect of spacing ratio at $Re = 150$ was done by Abbasi et al., (2014). They investigated four different types of flow mods depending on value of gap spacing. Lam and Zhou (2007) studied the flow behavior past over the four circular obstacles in an inline square arrangement at different values of g_c from 1.5 to 5 and **Reynolds number** was varied from 1.128×10^4 to 1.982×10^4 . The flow past over five inline circular obstacle experimentally examined by Hetz et al., (1991) at **Reynolds number** in the range $Re = 1 \times 10^4$ to 5×10^4 . They identified various flow structures at different values of Re and g with dominance of one flow mod over other. Sewatkar et al., (2012) numerically as well as experimentally investigated the flow around six inline square obstacles to study the flow mode at some selected values of g_c and Re . that are $0.5 \leq g_c \leq 10$ and $80 \leq Re \leq 320$. They investigated the negligible influence in average Cd_{mean} of first obstacle due to presence of other two obstacles. The flow past over six square obstacles in tandem configuration at the spacing range $1.5 \leq g_c \leq 15$ and $Re = 100$ is numerically examined by Bao et al., (2012) and six different types of flow mods were found in their investigations.

From above mentioned literature, it is cleared that no more attention was paid to study the flow past more than four objects in tandem arrangement. No work in literature was reported on the influence of gap spacing for five tandem square obstacles at constant value of Reynolds number i.e. $Re = 150$. By keeping focus on these points, the present study was based: (i) to investigate the effects of spacing ratio for the flow past five square obstacles aligned in-line at $Re = 150$ (ii) to examine the variation of flow mod in wake region and outside the wake region in the whole computational domain, (iii) to study the physical parameters with respect to change in spacing ratios and (iv) to study the applications of lattice Boltzmann method for flow past over the complicated fluid flow problem. These points will be discussed in the following sections in detail.

2. Lattice Boltzmann Method

Lattice Boltzmann method is a numerical approach used to solve simple or complex fluid flow problems over the last few decades. It was derived by LGCA (Mohammad (2011)) and firstly introduced in 1988 by Mac-Namara and Zanetti (1988). LBM has become a most important technique for solving fluid flow problems, due to its easy execution, parallel computing system and ability to handle complicated and complex geometries, it has become a prevalent technique for evaluating simple and complex fluid flow problems (Wolf-Gladrow (2005)). Additionally, the pressure is a local variable in Lattice Boltzmann Method and there is no use of Poisson equation to solve for finding the pressure (Mohammad (2011)). We can compute pressure through equation of state ($p = \rho c_s^2$). The LBM is based on the famous Lattice Boltzmann Equation (LBE):

$$h_i(\mathbf{x} + \mathbf{c}_i, t + 1) = h_i(\mathbf{x}, t) + \Omega_i(\mathbf{x}, t) \quad (1)$$

where $h_i(\mathbf{x}, t)$ is the density distribution function that indicates the position \mathbf{x} of a particle at time t , \mathbf{c} is the velocity of the particle and Ω is the collision operator. Using the Bhatnagar-Gross-Krook (BGK) collision operator, the discretized form of the above equation is given by Bhatnagar-Gross-Krook (1954),

$$h_i(\mathbf{x} + \mathbf{c}_i, t + 1) = h(\mathbf{x}, t) - 1/\tau[(h_i(\mathbf{x}, t) - h_i^{eq}(\mathbf{x}, t))] \quad (2)$$

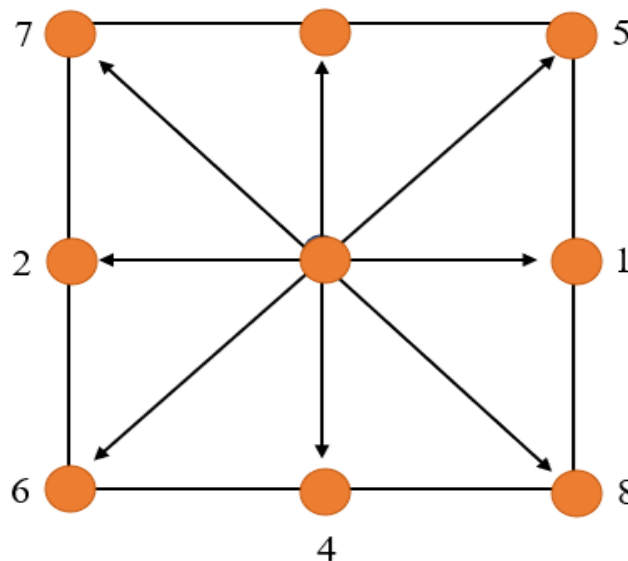


Fig 1. Two dimensional nine particle velocity model (d'Humieres (1992)).

Where τ is the single relaxation time and to control stability of method, whereas $h_i(\mathbf{x}, t)$ and $h_i^{eq}(\mathbf{x}, t)$ is density distribution and the equilibrium distribution function (EDF) given by Lam and Zhou (2007),

$$h_i^{eq} = \xi_i \eta \left[1 + \frac{\mathbf{c}_i \cdot \mathbf{u}}{c_s^2} + \frac{(\mathbf{c}_i \cdot \mathbf{u})^2}{2c_s^4} - \frac{\mathbf{u}^2}{2c_s^2} \right] \quad (3)$$

LBM based on different structure or model, but most commonly used model is D2Q9 (where D is represented the dimensions and Q represented the number of particles) (see Fig 1). We are also using this model for present problem. (d'Humieres (1992)). The weighting coefficients for D2Q9 model are, $\xi_r = 4/9$ for $r = 0$, $\xi_s = 1/9$ for $s = 1, 2, 3, 4$ and $\xi_l = 1/36$ for $l = 5, 6, 7, 8$, respectively. The discrete velocity set $\{\mathbf{e}_i\}$ is written as $\mathbf{e}_0 = (0, 0)$, $\mathbf{e}_{1,3} = (\pm 1, 0)$, $\mathbf{e}_{2,4} = (0, \pm 1)$, and $\mathbf{e}_{5,6,7,8} = (\pm 1, \pm 1)$. Moreover, $c_s = 1/\sqrt{3}$ is the speed of sound and used to find the kinematic viscosity, $\gamma = (\tau - 0.5)c_s^2$.

3. Geometry of problem and boundary conditions

The geometry of present problem is based on flow past five square obstacles aligned inline of similar sizes 'D' as shown in Fig 2. The values of gap spacing is varied from $g_c = 0.25$ to 5 at fixed value of $Re = 150$. Obstacle C_1 is the upstream obstacle placed at $L_u = 6D$ from the inlet position, while obstacle C_5 is the obstacle placed at last position. The distance from C_5 to exist position of channel is called downstream position is denoted by L_d and taken as $L_d = 25D$. While the other three obstacles C_2, C_3 and C_4 are put in between the upstream and downstream obstacles. The height of channel is fixed at $H = 11D$ and its length varies by varying the spacing ratios between the obstacles as shown in Table 1. Uniform inflow velocity condition ($\mathbf{u} = U_\infty$; $\mathbf{v} = 0$) is used at inlet, where U_∞ is inflow velocity along x- direction and $\mathbf{v} = 0$ is shown that there is no flow in y-direction. The convective boundary condition ($(\frac{\partial \mathbf{u}}{\partial t} + U_\infty \frac{\partial \mathbf{u}}{\partial x} = 0)$) is applied at the outlet position of the channel (Guo (2008)).

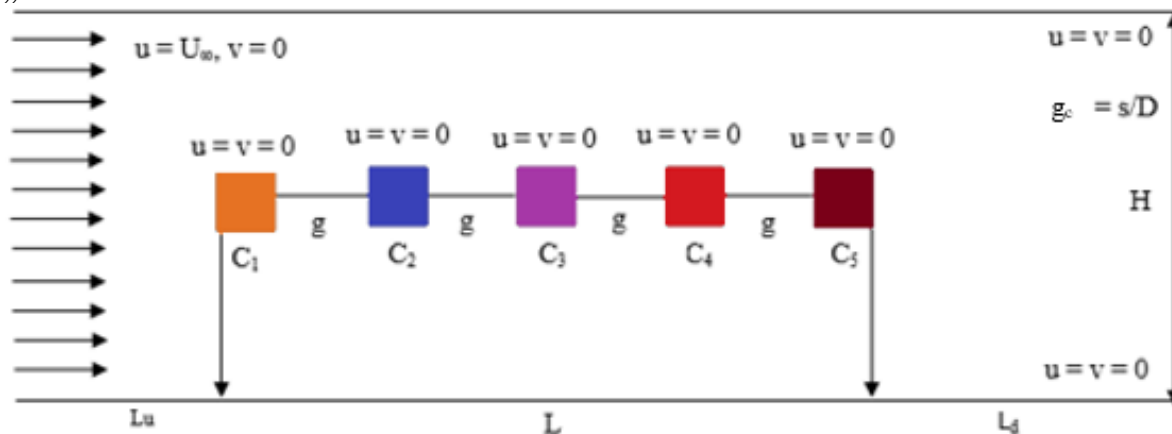


Fig 2. Schematic diagram for the flow past five in-line square obstacles.

Due to stationary position of obstacles, no-slip condition ($\mathbf{u} = \mathbf{v} = 0$) is also used at the walls of the obstacles as well at the top and bottom walls of the channel (Yu et al., (2003)). The total forces of fluid on the square obstacle are calculated through the momentum exchange method (Yu et al., (2003)). In the present problem by using the LBM, we used the value of uniform velocity $U_\infty = 0.04385964$ for getting reasonably good results at $\tau = 0.526315784$ for $Re = 150$ and $D = 20$. The code for present study was written and edited using Compaq Visual Fortran version 6.5.0. All selected cases for present simulation are shown in Tab. 1.

Table 1. Selected cases for simulation of present study at $Re = 150$.

Cases	$L \times H$	Cases	$L \times H$
$g_c = 0.25$	741×221	$g_c = 2.5$	921×221
$g_c = 0.5$	761×221	$g_c = 2.75$	941×221
$g_c = 0.75$	781×221	$g_c = 3$	961×221
$g_c = 1$	801×221	$g_c = 3.25$	981×221
$g_c = 1.25$	821×221	$g_c = 3.5$	1001×221
$g_c = 1.5$	841×221	$g_c = 3.75$	1021×221
$g_c = 1.75$	861×221	$g_c = 4$	1041×221
$g_c = 2$	881×221	$g_c = 5$	1121×221
$g_c = 2.25$	901×221		

4. Code validation and grid independence study

In order to find the adequate grid points for present problem, the developed code is checked by considering 10D, 16D, 20D, 24D, 30D and 40D points along the surface of obstacle. The physical parameter Cd_{mean} , Cd_{rms} , Cl_{rms} and S_f are computed at $Re = 150$ as shown in Tab 2. $X \times Y = 321 \times 111$ points are used in x, y directions, respectively, for 10D, 513 \times 177 points used for 16D, 641 \times 221 points used for 20D, 768 \times 265 points used for 24D, 961 \times 331 for 30D and 1281 \times 441 used for 40D, respectively in x, y directions. It is noticed from the table that the data obtained from the 20D grid points gives accurate results to meet convergence criteria as compared to 10D, 16D, 24D, 30D and 40D grid points. Therefore, whole simulations are done by using 20D grid points for present problem.

Table 2. Comparison of physical parameters at various grid points for $Re = 150$.

Re = 150	10D	16D	20D	24D	30D	40D
$C_{d_{mean}}$	1.307 (2.43%)	1.276 (0.08%)	1.275 (1.03%)	1.262 (0.56%)	1.255 (3.46%)	1.213
$C_{d_{rms}}$	0.0199 (19.2%)	0.0167 (3.73%)	0.0161 (5.9%)	0.0152 (4.11%)	0.0146 (6.6%)	0.0137
Cl_{rms}	0.304 (12.1%)	0.271 (2.65%)	0.264 (2.32%)	0.258 (1.18%)	0.255 (1.63%)	0.2509
S_t	0.1756 (8.7%)	0.1615 (1.44%)	0.1592 (6.5%)	0.1494 (6.9%)	0.1369 (1.94%)	0.1343

In order to validate our numerical code for current study, we have calculated the values of physical parameters like $C_{d_{mean}}$ and S_t for flow past a SSR (Single square obstacle) at $Re = 100, 150, 175$ and 200 . The numerical technique is validated by comparing the obtained results from current simulation with other available data in literature for the flow around a SSR as described in Table 3, where Exp representing the experimental work and num is representing the numerical work.

Table 3. Study of code validation for flow past a SSR at $Re = 150$

Re	$C_{d_{mean}}$				S_t			
	100	150	175	200	100	150	175	200
Present	1.484	1.493	1.524	1.572	0.149	0.154	0.165	0.149
Okajima (1982) exp	1.60	1.492	1.482	1.480	0.141	0.142	0.140	0.138
Davis & Moore (1982) exp	1.64	-----	-----	1.72	0.1482	0	-----	0.1470
Malikzadeh & Sohankar (2012) num	1.444	1.408	1.412	1.424	0.145	0.161	0.165	0.163
Robichaux et al., (1999) num	1.54	1.56	1.60	1.63	0.154	0.164	0.158	0.156
Gera & Sharma (2010) num	1.461	1.411	1.472	1.487	0.129	0.141	0.143	0.143
Abograish & Alshayii (2013) num	1.485	1.474	1.483	1.488	0.140	0.1528	0.1528	0.1528

It can be examined that $C_{d_{mean}}$ and values of Strouhal number are shown a good agreement with the experimental as well as numerical values available in literature. This indicated that the present code can simulate the flow characteristics in an efficient manner. There may be observed some minute differences in between the calculated values and available data from literature at $Re = 100, 175$ and 200 due to use of different numerical techniques, boundary conditions as well use of different grid points. The comparison of $C_{d_{mean}}$ for flow past two, three, four and five obstacles aligned inline is described in Tab. 4.

Table 4. Comparison of $C_{d_{mean}}$ for flow past two, three, four and five obstacles aligned inline.

Single Obstacle	Two inline arrangement					
	Cd _{mean}	Cd _{mean} 1	Cd _{mean} 2	-----		
	1.340	1.2892	1.0126	-----		
% Reduction Cd _{mean}		3.8%	24.43%	-----		
	Three inline arrangement					
	Cd _{mean}	Cd _{mean} 1	Cd _{mean} 2	Cd _{mean} 3	-----	
	1.340	1.2503	-0.0836	0.2184	-----	
% Reduction Cd _{mean}		6.7%	106.23%	83.7%	-----	
	Four inline arrangement					
	Cd _{mean}	Cd _{mean} 1	Cd _{mean} 2	Cd _{mean} 3	Cd _{mean} 4	
	1.340	1.491 (49.1%)	1.123 (16.19%)	0.093 (93.1%)	0.122 (87.8%)	
% Reduction Cd _{mean}	Five inline arrangement					
	Cd _{mean}	Cd _{mean} 1	Cd _{mean} 2	Cd _{mean} 3	Cd _{mean} 4	Cd _{mean} 5
	1.340	1.454	0.8546	0.0364	0.1115	0.2203
% Reduction Cd _{mean}		8.5%	36.2%	97.3%	91.7%	83.6%

5. Results and discussions

A (2-D) numerical study is carried out for flow past five inline square obstacles of same sizes (D) at constant value of $Re = 150$. The gap spacing is chosen between the obstacles from $g_c = 0.25 - 5$. The flow structure mechanism is analyzed in terms of vorticity, time-history analysis of drag and lift coefficients and force statistics.

5.1. Analysis of Vorticity contour and drag and lift coefficients

In this section, the existing flow mods from $g_c = 0.25 - 5$ and their drag and lift coefficients are discussed briefly at $Re = 150$ for sake of simplicity and repeated results are avoided to show here.

5.1.1. Share layer reattachment flow mod (SLR)

This flow mod is observed at $g_c = 0.25-1.75$ as shown in Fig 3(a-d). Where the generated shear layer after hitting the obstacle, separated into two layers, first one is moved to upside of the obstacle and other one moved towards the downside of the obstacles. These two layers are reattached directly at downstream location of channel after splitting from obstacles, so called SLR flow mod. At $g_c = 0.25$ and 0.5 , due to narrow space, no flow interaction is observed in between the obstacles, but by increasing g_c from 0.75 to 1.75 , symmetric flow behavior is found in between the gap. An alternate vortex shedding is examined towards the downstream of obstacles in the form of von-Karman Vortex Street. A slight modulation in the flow behavior is occurred at $g_c = 1.75$ as compared to $g_c = 0.25 - 1.5$ due to this recirculation zone observed behind the obstacle C_4 . The shape of vortices became more prominent with large gap spacing and there is a variation in the shape and size of the vortices with increase in gap spacing. The width of wake region also varied to some extent with increment in values of gap spacing.

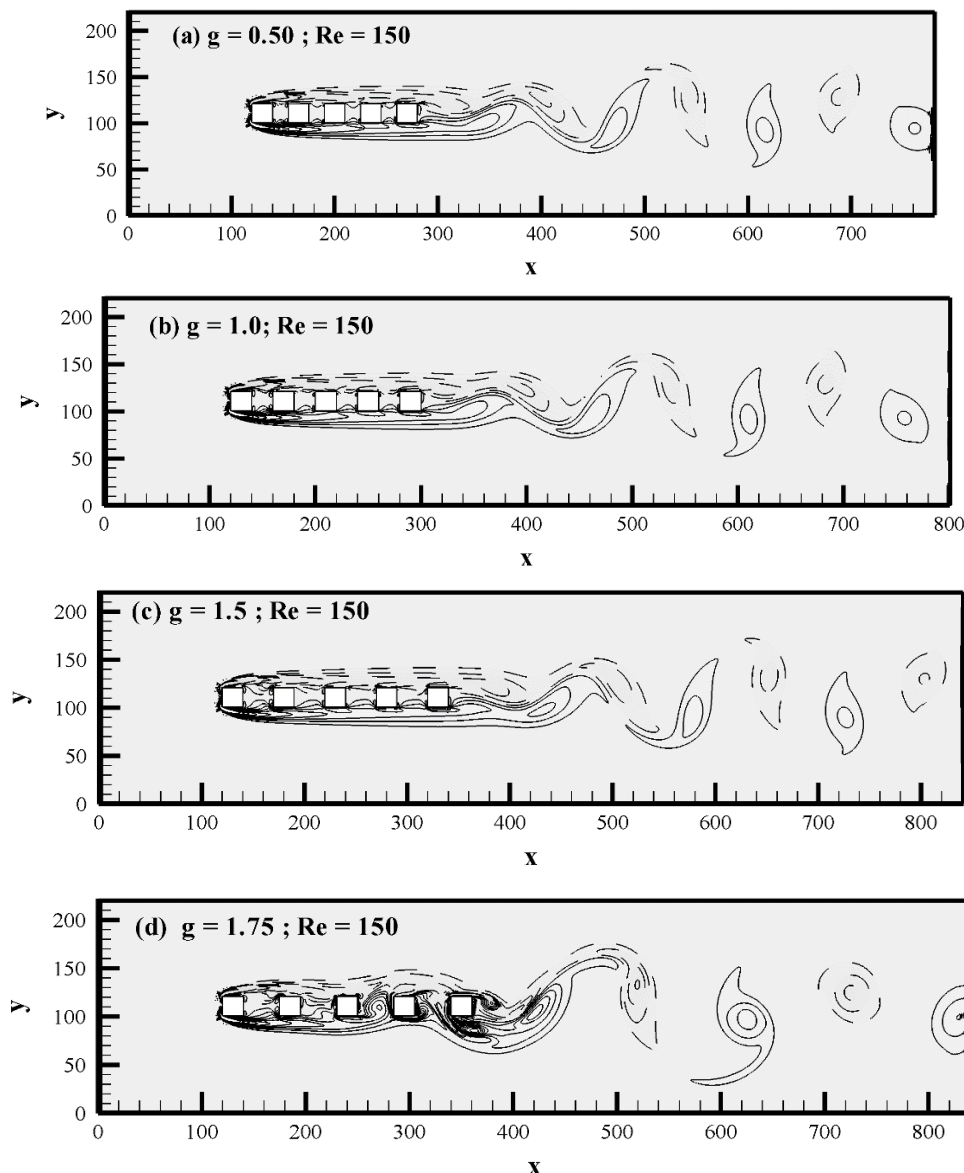


Fig 3(a-d). Vorticity contour for SLR flow mod.

The drag coefficient of fluid force is confirmed that there is no flow existed between the obstacles and constant drag force are exerted by the flow on obstacles as shown in Fig 4(a-h). It can be noticed that drag force of obstacle C_2 is negative. This is due to the fact, that the spacing between the obstacles is narrow and strong suction is created in the gaps. As a result, drag force became negative. The magnitude of C_d for obstacle C_1 is maximum and minimum for obstacle C_2 . The lift coefficients is represented the periodic behavior due to an alternate vortex shedding at downstream location and the behavior of vortices remain almost same at downstream location of obstacles (see Fig. 3). The amplitude of C_l is increased with the increment in gap from $g_c = 0.25$ to 1.75 . In comparison of all five selected obstacles, obstacle C_5 having largest and obstacle C_1 is contained smallest amplitude of lift coefficients.

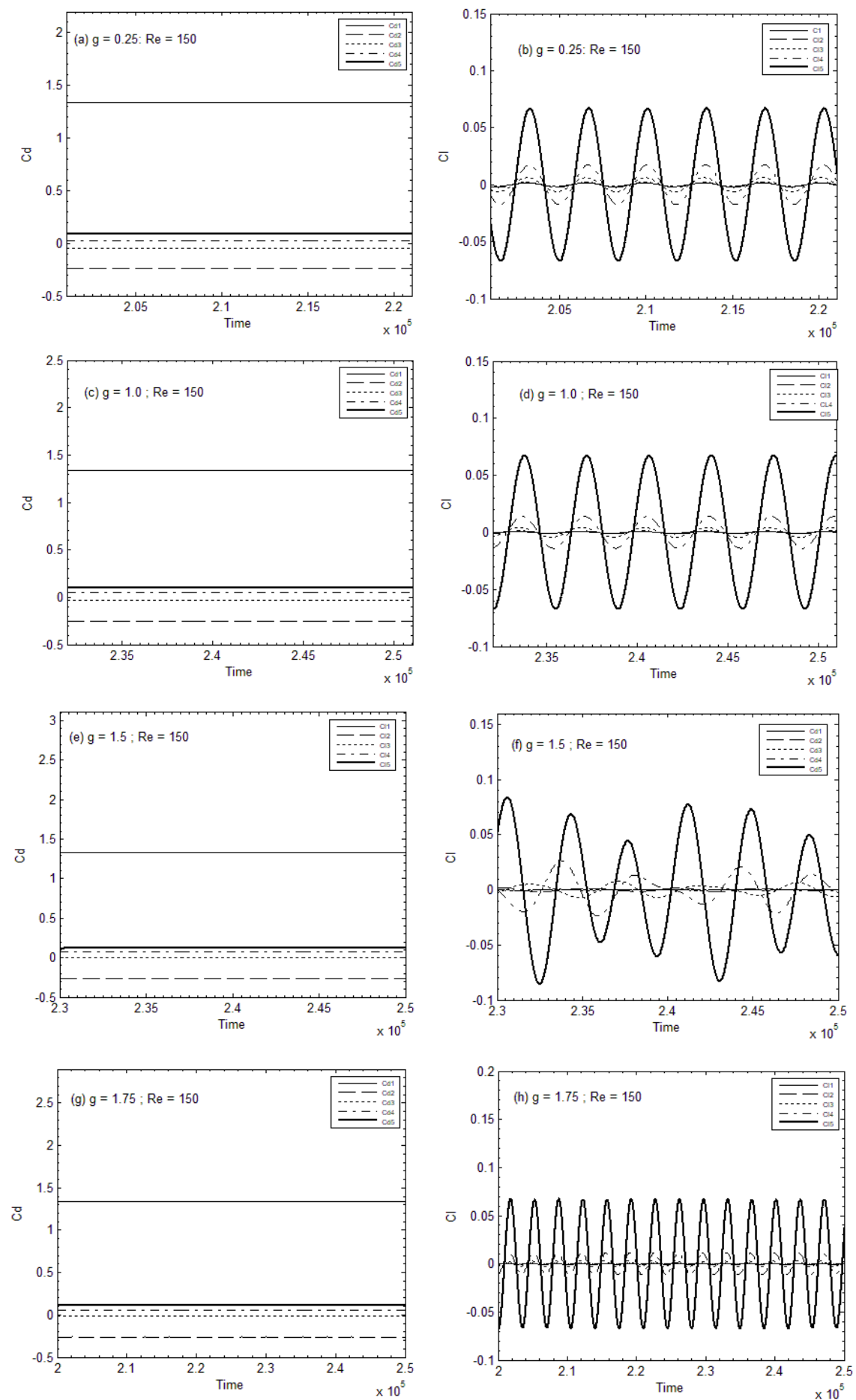


Fig 4(a-h). Drag and lift coefficients for SLR flow mod.

5.1.2. Half-developed irregular vortex shedding flow mod

The second existing flow mod is named as HDIVS (Half developed irregular vortex shedding) flow mod, observed only at $g_c = 2.0$ and 2.25 at $Re = 150$. In this flow mod, no flow is passed in between the gap of obstacles C_1 to C_2 , and shear layers between the obstacles C_2 to C_5 , just started to roll up in order to generate vortex shedding. However, the vortices are not generated completely in between the gap. The shed vortices, scattered from the rear ends of obstacles are shown irregularity in behavior and thus failed to form Von-Karman Vortex Street at downstream of channel. The distortion of vortices behind the obstacles clearly observed in Fig. 5(a, b). Due to incomplete generation of vortex shedding in between the obstacles, we named this flow mod as half developed irregular vortex shedding flow mod. Moreover, the vortices are no more round in shape as compared to Shear layer flow. With increase in gap spacing, strength and size of vortices also varied.

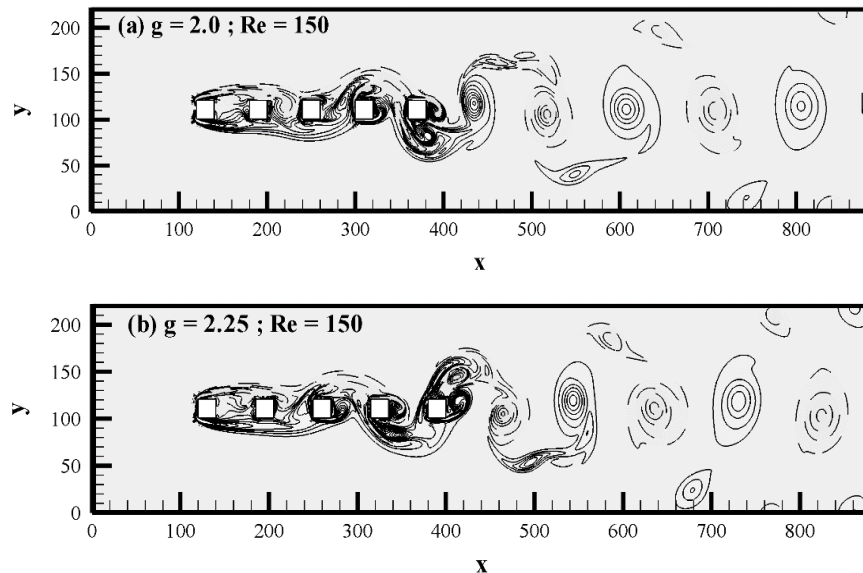


Fig 5(a-b). Vorticity contour for HDIVS flow mod.

The drag coefficients for HDIVS flow mod at $g_c = 2.0, 2.5$ at $Re = 150$ is showing the periodic behavior (see Fig. 6). The magnitude of C_{d1} is smallest than that of drag coefficients of other selected obstacles. It is also seen from the Figure that C_2 having negative drag coefficient. The lift coefficients of obstacles C_1 and C_2 are contained periodic behavior but for obstacles C_3 to C_5 , lift coefficient is modulated due to an irregular vortex shedding. The maximum amplitude of lift coefficient is found for obstacle C_5 and minimum for obstacle C_1 .

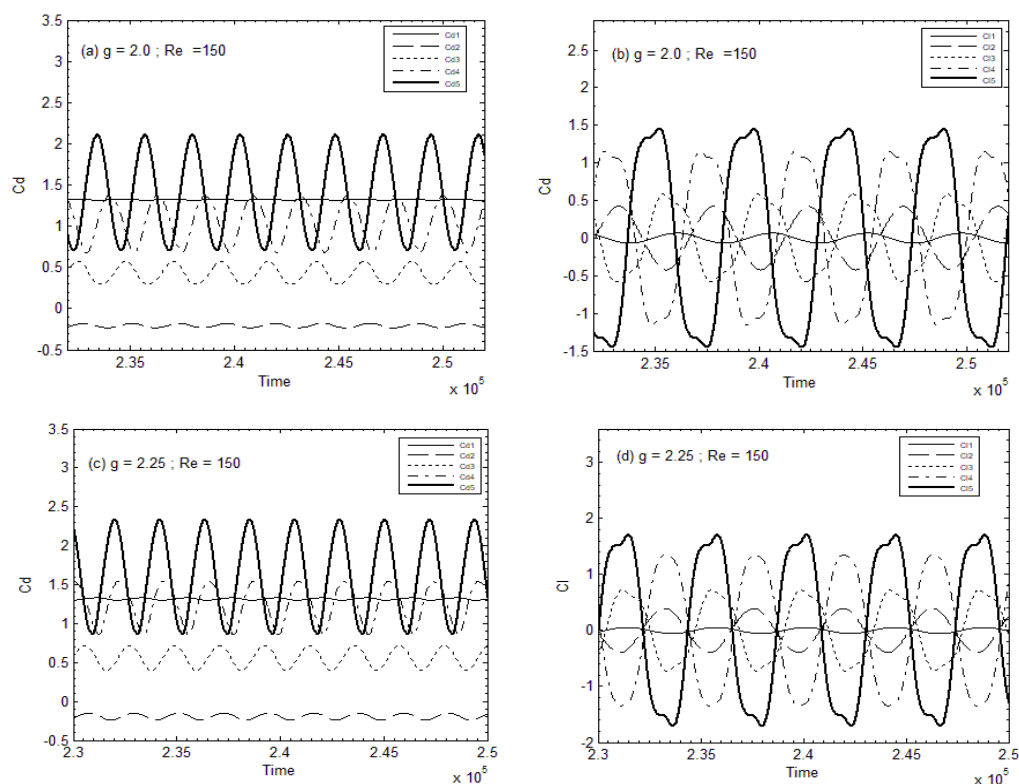


Fig 6(a-d). Drag and lift coefficients for HDIVS flow mod.

5.1.3. Fully developed two rows vortex shedding flow mod

The other existing flow mod is named as FDTRVS (fully developed two rows vortex shedding) flow mod observed at $g_c = 2.5, 2.75, 3, 3.25$ and 3.5 for fixed Reynolds number i.e. $Re = 150$ as shown in Fig. 7(a, b). In this flow mod, vortices are fully developed in between the gap of obstacles C_1 - C_5 as well towards the down position of channel. The shed vortices behind the obstacle C_4 , are changed into two rows after hitting the fifth obstacle. These two rows moved parallel to each other. Therefore, we called it fully developed two rows vortex shedding FDTRVS flow mod. When the indicated distance is less, the frequency generation of vortices is high and effected on size of vortices. Moreover the distance between two parallel rows of vortices at downstream of obstacles is decreased with increment in gap is represented in Fig. 7(c).

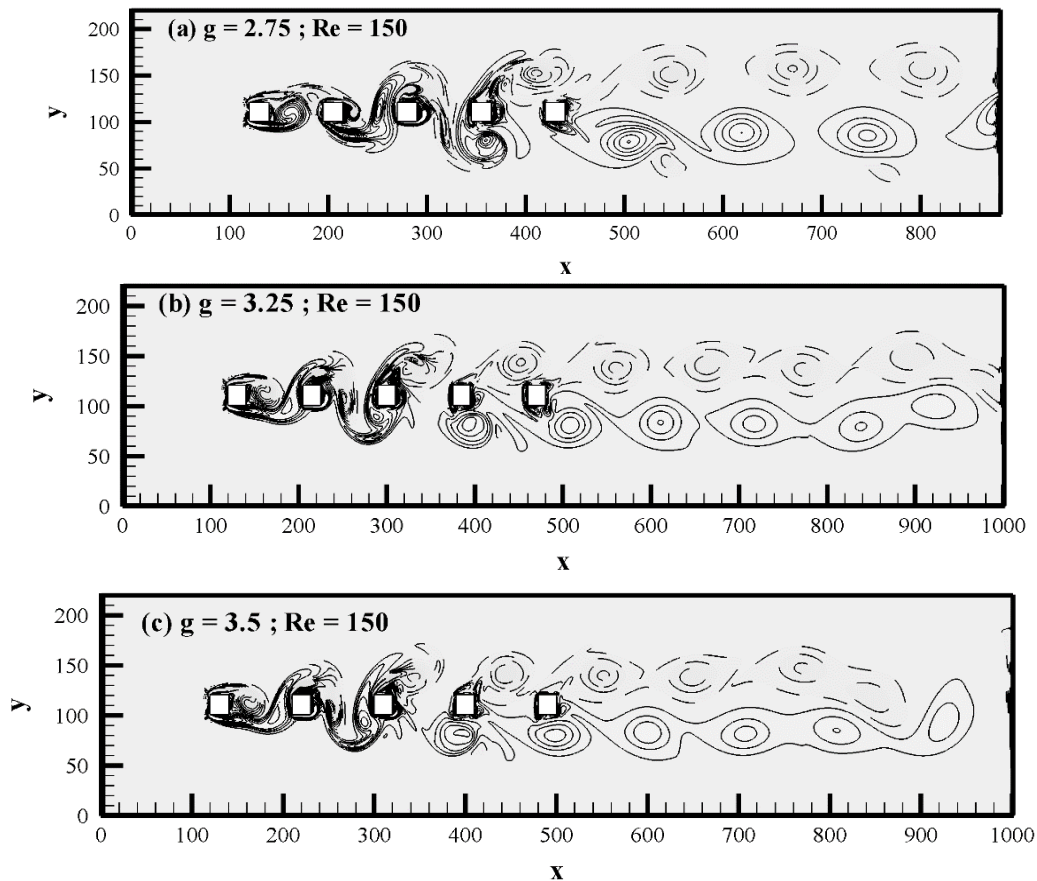
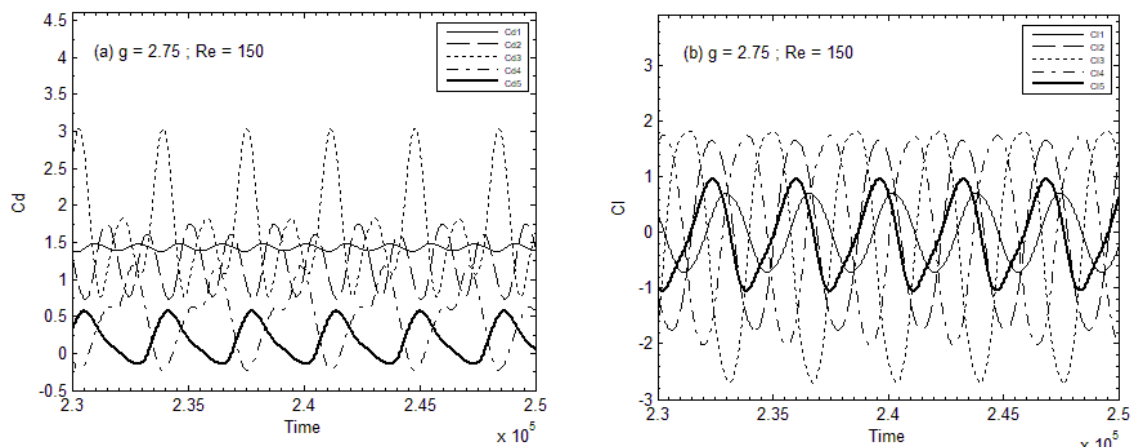


Fig 7(a-c). Vorticity contour for FDTRVS flow mod.

The drag coefficient at $g_c = 2.75, 3.25$ at $Re = 150$ for FDTRVS flow mod is shown in Fig 8(a-d). It is observed that drag coefficients is shown almost the periodic behavior due to formation of fully developed vortices between the obstacles except for drag coefficient of obstacle C_4 at $g_c = 2.75$, which contained modulated behavior due to critical shape and size of vortices generated by obstacle C_4 . The magnitude of drag coefficient for obstacle C_2 is maximum and the magnitude of obstacle C_5 is minimum for all observed case.



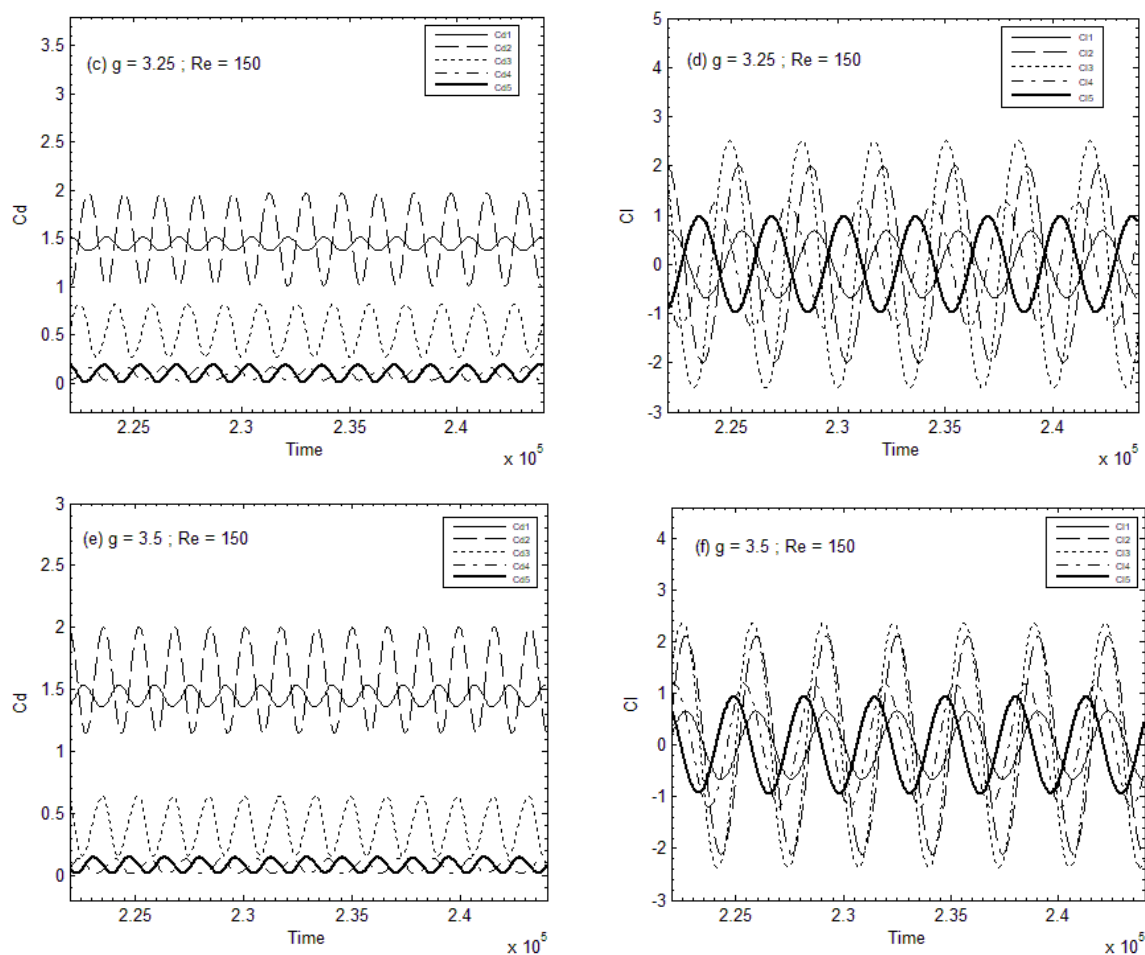
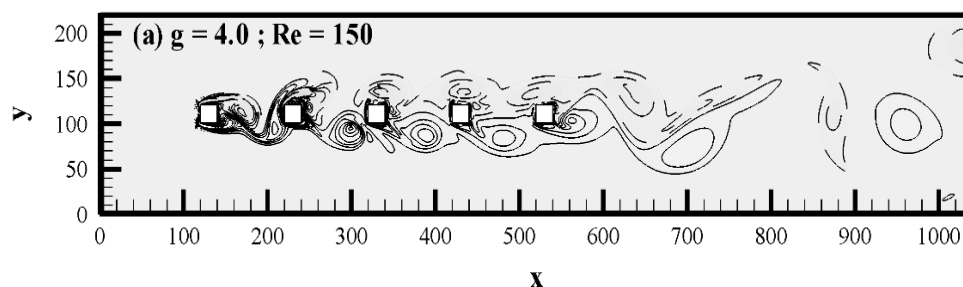


Fig 8(a-f). Drag and lift coefficients for FDTRVS flow mod.

The lift coefficients are showing periodicity, due to formation of regular vortex shedding at downstream of obstacles and their amplitude remained almost same with increase in gap spacing due to similar behavior of vortices. The maximum magnitude of lift coefficients is observed for obstacle C_3 and minimum for obstacle C_1 .

5.1.4. Two rows single bluff body flow mod

The last flow mod is existed at large gap spacing i.e. $g_c = 3.75, 4$ and 5 at $Re = 150$. In this flow mod, shear layer rolled up in between the gap of obstacles C_1-C_2 and an alternate vortex shedding is found between the obstacles C_1-C_2 and C_2-C_3 , but from obstacles C_3-C_5 , an alternate vortex shedding is transformed into two rows vortex shedding after hitting obstacle C_3 . At some extent, these two rows moved parallel to each other, with opposite signs of shed vortices in the near wake. Moreover, the two parallel vortex rows did not persist for a long distance and started to merge with each other in form of single row at a downstream position of channel. We called this flow behavior as TRSBB (two rows single bluff body) (see Fig 9(a-b)). There is an irregularity in the shape and size of vortices. The size of vortices are decreased and strength increased with an increasing in the gap spacing.



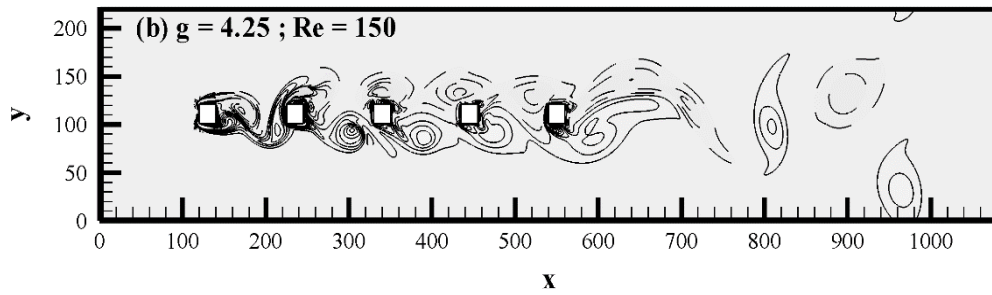


Fig 9(a-b). Vorticity contour for TRSBB flow mod.

The magnitude of drag coefficients for two rows single bluff body is periodic for the obstacles C_1 - C_4 , except for obstacle C_5 whose drag coefficient is modulated due to complicated structure of vortices generated at the end of obstacle C_5 as clear from Fig 10(a-d). It is noted that the amplitude for C_{d1} is larger and that of C_{d3} is smallest as compared to the C_d of other three remaining obstacles. The magnitude of C_l is periodic for all selected five obstacles due to same behavior of vortices at downstream position of obstacles. The obstacle C_2 is contained maximum value of lift coefficient and obstacle C_1 contained smallest value. The all existing flow mods at different values of g_c at $Re = 150$ are shown in Table 5.

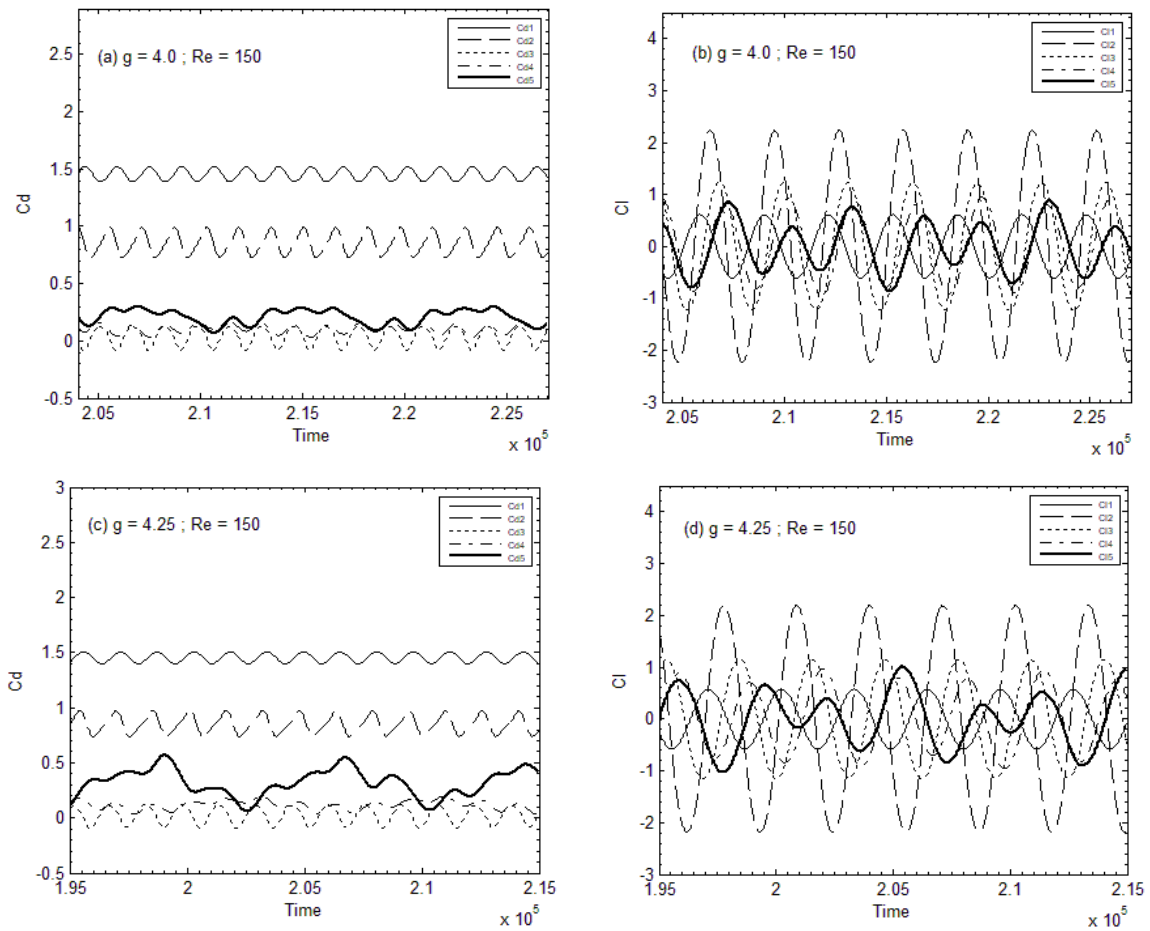


Fig 10(a-d). Drag and lift coefficients for TRSBB flow mod.

Table 5. Existing flow mods at $g_c = 0.25 - 5$

Gap spacing	Flow mods
$g_c = 0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75$	Shear layer reattachment flow mod (SLR)
$g_c = 2, 2.25$	Semi-developed irregular vortex shedding flow mod (FDIVS)
$g_c = 2.5, 2.75, 3, 3.25, 3.5$	Fully developed two rows vortex shedding flow mod (FDTRVS)
$g_c = 3.75, 4, 5$	Two rows single bluff body flow mod (TRSBB)

5.2. Force statistics

In physical parameters, the values $C_{d_{mean}}$, $C_{d_{rms}}$, $C_{l_{rms}}$ and S_t are calculated against the various values of g with fixed values of $Re = 150$ (see Fig. 11(a-d)). The values of $C_{d_{mean}}$ are represented in Fig. 11(a). The mean drag force of obstacle C_1 and C_2 having similar behavior. Their values are increased from $g_c = 0.25$ to 0.5 , then declined in their values observed from $g_c = 0.5$

till 2.5. After that from $g_c = 3 - 5$, an increased in values of $C_{d_{mean}}$ for obstacles C_1 and C_2 is occurred. An opposite behavior for obstacles C_3-C_5 is observed from the obstacles C_1 and C_2 . The mean drag force values for C_3-C_5 is decreased from $g_c = 0.25 - 0.5$ and after that their values started to be increased from $g = 0.5 - 3$. At $g > 3$, mixed trend is observed for $C_{d_{mean}}$ of obstacles C_3-C_5 . The highest value of mean drag force is found at $g = 5$ for obstacle C_1 and it is 1.4594, while smallest value of $C_{d_{mean}}$ is noticed for obstacle C_3 at $g_c = 0.25$. There may also seemed some negative values of $C_{d_{mean}}$ for obstacle C_2 at $g_c = 0.25 - 2.25$ and for obstacle C_3 at $g_c = 0.25 - 1.25$ due to effect of thrust. The values of $C_{d_{rms}}$ and $C_{l_{rms}}$ against gap spacing are represented in Fig. 11(b, c) and they are representing mixed trend by increasing the values of gap spacing for all five selected obstacles. The largest value of $C_{d_{rms}}$ reported at $g_c = 5$ for obstacle C_1 , i.e. 0.7102, where existing flow mod is SBB flow mod. The smallest range of $C_{d_{rms}}$ is investigated at $g_c = 0.25$ for C_2 and that is 0.00011. Here the existing flow mod is Shear layer flow mod. Similarly the maximum and minimum values for root men square values of lift coefficients are 1.6976 at $g_c = 2.75$ for obstacle C_3 and 0.00037 at $g_c = 1.25$ for obstacle C_1 , respectively. Fig. 11(d) is drawn to represent the Strouhal number against gap spacing for obstacles C_1-C_5 . The S_t values from $g_c = 0.25 - 0.75$ are increasing and then decreasing at values of $g_c = 1 - 1.5$. As gap spacing is increased from $g_c = 1.75 - 5$, the Strouhal number is again shown an increasing behavior continually. The greatest value of Strouhal number is examined at $g_c = 5$ for obstacle C_5 , i.e. 0.1521. Where the flow mod is TRSBB. The smallest value of S_t is obtained at $g_c = 1.5$ for obstacle C_3 and that is 0.0862.

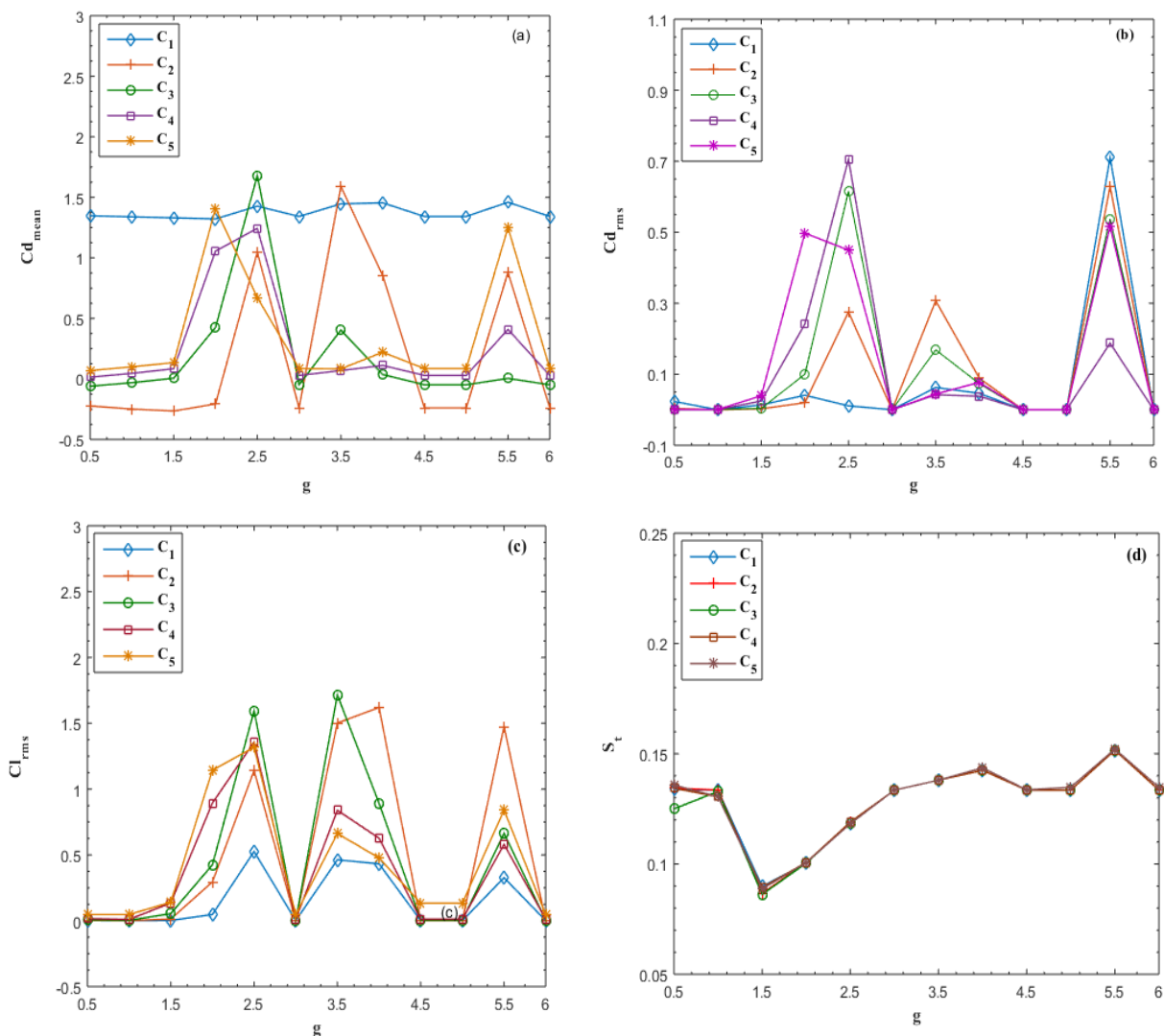


Fig 11. Change in values of (a) $C_{d_{mean}}$ (b) $C_{d_{rms}}$ (c) $C_{l_{rms}}$ and (d) S_t with spacing $g_c = 0.25-5$ at $Re = 150$.

6. Conclusions

Two dimensional numerical simulation are conducted to find the effect of spacing ratio on variation of flow characteristics within the wake region and behind of the wake region in whole computational domain. This study was also conducted to examine the influence of gap ratios on force statistics for the flow around five aligned inline square obstacles of similar sizes. The numerical method used was the LBM. First of all the code validity was checked for the flow past a SS obstacle and obtained results were compared with results, those were available in literature, either experimental or numerical. Similar behavior were found among all the results. After that, numerical calculations were done for flow past five square obstacles placed in tandem arrangement. Important results obtained from this study are given below:

1. Four distinct kinds of flow mods were observed at $g_c = 0.25$ to 5 in this study. These are (a) Shear layer flow mod, (b) Hemi-developed irregular vortex shedding flow mod, (c) Fully developed two rows vortex shedding flow mod, (d) Two rows single bluff body flow mod.

2. The behavior of $C_{d_{mean}}$ for obstacles C_1 and C_2 is quite different from obstacles C_3 - C_5 . The greatest value of $C_{d_{mean}}$ is found at $g_c = 5$ for obstacle C_1 and it is 1.4594, while the minimum value of $C_{d_{mean}}$ is for obstacle C_3 noted at $g_c = 0.25$.
3. There may also existed some negative values of $C_{d_{mean}}$ for obstacle C_2 at $g_c = 0.25$ to 2.25 and for obstacle C_3 at $g_c = 0.25$ to 1.25 due to effect of thrust.
4. The largest range of $C_{d_{rms}}$ and $C_{l_{rms}}$ are obtained at $g_c = 5$ for obstacle C_1 and at $g_c = 2.75$ for obstacle C_3 , respectively.
5. The largest range of S_t is examined at $g_c = 5$ for obstacle C_5 and the smallest value of S_t is occurred at $g_c = 1.5$ for obstacle C_3 and that is 0.0862.

Conflict of Interest

The author declares no conflict of interest.

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Not available

Data Availability Statement

Not applicable

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