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A Two-Dimensional Instability In Mixed Convection With Spatially Periodic Lower Wall Heating By LBM Method

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ABSTRACT
Numerical study for 2-D mixed convection (MC) flow at bottom heated wall has been carried out through lattice Boltzmann method (LBM) using periodic boundary condition. The uniform temperature is applied at top surface by considering zero temperature and velocity conditions initially throughout the channel. This study based to compare the results of flow structure obtained by using numerical techniques i.e. lattice Boltzmann method under the influence of non-dimensional parameters such as Rayleigh (Ra), Prandtl (Pr) and Reynolds numbers (Re) on vertex flow characteristics and its stability behavior. Different flow structures have been observed by using different ranges of Re and Ra numbers, while keeping Pr number fix (i.e. Pr = 1). It is investigated that at some range of Re and Ra, the results obtained are in good agreement with Tangborn results. Furthermore, it is investigated that the separation vortex generated at downstream of hot wall has point of inflection in flow, which leads towards instability and its motion is representing an unstable travelling wave (stream wise instability) behavior at high range of Re and Ra numbers. The results are shown in terms of streamlines, isotherms and time-trace analysis of inflow velocity.

Keywords: Lattice Boltzmann method, Mixed Convection, Streamlines, Isotherms, Instability.

Introduction

Bouncy driven secondary convection in a forced laminar flow may lead to heat transfer enhancement and the onset of turbulence. The understanding of thermal and flow characteristics is this so-called mixed convection. It has great significance in design of compact heat exchangers and chemical vapor deposition reactors or in the cooling of microelectronic equipment. Many theoretical, numerical and experimental studies have been carried out to investigate mixed convection (Luijkx et al., 1981; Chiu and Rosenberg, 1987; Chui et al., 1987; Evans and Grief, 1989; Ouazzani et al., 1990).

Ostrach and Kamotani (1975), studied mixed convection experimentally in a horizontal gap and forced convection was superimposed on natural convection with uniform wall temperature. It was investigated that steady vortex rolls aligned to flow direction are dominant structures at $Ra < 8000$ and $10 < Re < 100$. Ostrach observed the same structure unstable and an unsteady motion with no regular vortex structure appeared at $Ra > 8000$. Unsteady behavior of flow was also found by Osborne and Incropera (1985) experimentally on same geometry that has been used by Ostrach at higher Reynold and Rayleigh numbers. The resulting flow is separated just downstream of heat source. So that a cross-stream vortex roll is created that is opposite to the longitudinal rolls found in the uniform heating case.

Linear stability calculations in plane channel flow were done by Orszag (1971), he predicted that the laminar state becomes unstable at a critical Reynolds number $Re = 5772.22$. Michael et al. (1995) performed laboratory experiments and numerical computation to find instability in a spatially periodic open flow. They demonstrated that the existence of stable flow states with low-dimensional dynamics arising from primary and secondary transitions in a spatially periodic channel flow, two dimensional waves are stable for the range of Re above the primary transition at Re=130, and stable three-dimensional waves arise from secondary instability at $Re \approx 160$.
There has been particular interest in the structures of resulting flows and mechanism through which instability occurs. Relevant work is done by Ghaddar et al. [1986a; 1986b], on grooved channel flow, where separation occurs in grooved and separation region begins to oscillate and travelling waves appear at $Re_{cr} = 975$. An important aspect of that work is the supercritical flow oscillation at same frequency as unstable Tollmien-Schlichting (TS). Two dimensional steady numerical calculations on mixed convection with localized heating on lower and upper surfaces were carried out by Kennedy and Zebib (1983), where the flow separation occurs just downstream of heat source and vortex roll is now cross-stream. Due to large driving forces, flow becomes unstable. Wang et al. (1991) and Tangborn (1992), carried out the numerical simulation of mixed convection with

spatially periodic heating. Main difference between their work is that the Wang identified a space region (Re, Ra) where the flow was unsteady. While Tangborn (1992) investigated that the flow is stabilized until the upper wall moves. Further he investigated that the flow becomes unsteady at large value of Reynold number i.e. Re=100. This paper is based to find the critical value of parameters (Re and Ra) where the flow becomes unstable, to find wave structure of flow by using lattice Boltzmann method and comparison of our results with Tangborn (1992), who carried out numerical simulation of two-dimensional mixed convection flow. Furthermore, it will be investigated that the instability occurs is same to well-studied inviscid shear instability, the grooved channel flow or not.

Problem Description

Consider two-dimensional periodic channel flow with aspect ratio (L/l), L is the periodicity of channel and l is half height of channel. No slip boundary condition is applied at the walls: the upper wall is insulated while spatially periodic boundary condition is applied at lower wall that is $\sin(2\pi nx/L)$ where n=1,2,3...Flow is fully developed in stream wise (x) direction.
The governing equations in non-dimensional forms are

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \quad (1)$$

$$\frac{\partial u^*}{dt^*}+u^*.\nabla u^*=-\nabla p^*+\frac{1}{Re}\nabla^2 u+\frac{Ra}{Re^2}T^* \quad (2)$$

$$\frac{\partial T^*}{dt^*}+u^*.\nabla T^*=\frac{1}{RePr}\nabla^2 T^* \quad (3)$$

Boundary Conditions

$$v^*=0 \quad \text{at } y^*=\mp 1 \quad (4)$$

$$T^*=0 \quad \text{at } y^*=1 \quad (5)$$

$$T^*=\sin\left(\frac{2\pi mx}{L}\right) \quad \text{at } y^*=-1 \quad (6)$$

Above equations are nondimensionalized by using reference velocity U and h as a reference length, where L is the periodicity of channel. Reynold Re, Prandtl Pr, and Rayleigh number Ra can be defined as

$$Re=\frac{Ul}{\nu}, \quad Ra=\frac{g\beta\Delta t l^3}{\alpha\nu}, \quad Pr=\frac{\nu}{\alpha}$$

ν is kinematic viscosity , α is thermal diffusibility coefficient, β is thermal expansion coefficient and g is the gravity.

Numerical Method

The numerical method that we employed here, is lattice Boltzmann method (LBM) for flow field and temperature field in space and time coordinates.

LBM for flow field

$$f_i(x+e_i\Delta t, t+\Delta t)=f_i(x,t)+\frac{\Delta t}{\tau_v}\left[f_i^{eq}(x,t)-f_i(x,t)\right]+3w_i\nabla t\, c_i.F \quad (7)$$

In above equation ∇t denotes lattice time step, e_i is the discrete lattice velocity in i-th direction, τ_v denotes the lattice relaxation time for the flow field, $f_i(x,t)$ is the distribution function and $f_i^{eq}(x,t)$ is equilibrium distribution function corresponding to distribution function $f_i(x,t$, F representing here the external force. Where;

$$F=\frac{Ra}{Re^2}\theta^*$$

The equilibrium distribution function $f_i^{eq}(x,t)$ can be defined as

$$f_i^{(eq)}=w_i\rho\left[1+\frac{3(\mathbf{e}_i.\mathbf{u})}{c_s^2}+\frac{9(\mathbf{e}_i.\mathbf{u})^2}{2c_s^4}-\frac{3\mathbf{u}^2}{2c_s^2}\right] \quad (8)$$

ϱ is density, u is macroscopic velocity, $c_s=\frac{1}{\sqrt{3}}$ is the speed of sound and w_i are the weighting coefficients given by

$$w_0=\frac{4}{9}, \quad w_{1,2,3,4}=\frac{1}{9}, \quad w_{5,6,7,8}=\frac{1}{36} \quad (9)$$

On a two-dimensional square lattice with axes movement, diagonal movement and rest particles (called D2Q9) where D is the dimensions and Q are the number of particles is given by

$$\mathbf{e}_i=\begin{cases}(0,0) & i=0,\\ \left[\sin\left(\frac{i\pi}{2}\right),-\cos\left(\frac{i\pi}{2}\right)\right] & i=1,2,3,4,\\ \sqrt{2}\left[\cos\left(\frac{i-5\pi}{2}+\frac{\pi}{4}\right),\sin\left(\frac{i-5\pi}{2}+\frac{\pi}{4}\right)\right] & i=5,6,7,8.\end{cases} \quad (10)$$

The density ρ and macroscopic velocity u from the distribution function can be calculated as

$$\rho(\mathbf{x},t)=\sum_i f_i(\mathbf{x},t), \quad (11)$$

$$\rho(\mathbf{x},t)\mathbf{u}(\mathbf{x},t)=\sum_i f(\mathbf{x},t)\,\mathbf{e}_i, \quad (12)$$

While continuity and momentum equation can be recovered with pressure $p=c_s^2\rho$ and kinematic viscosity $\nu=\frac{2\tau_v-1}{6}$ by using Chapman-Enskog multi-scale expansion.

LBM for temperature field

The lattice Boltzmann model for temperature field is given as,

$$g_i(x + e_i\Delta t, t + \Delta t) = g_i(x, t) + \frac{\Delta t}{\tau_t}\left[g_i^{eq}(x, t) - g_i(x, t)\right] \qquad (13)$$

Where $g_i(x, t)$ is temperature distribution function , $g_i^{eq}(x, t)$ is equilibrium distribution function corresponding to temperature distribution function in lattice velocity e_i, τ_t is relaxation time related to thermal diffusibility α, defined in terms of its respective relaxation time i.e. $\alpha = c_s^2(\tau_t - 0.5)\nabla t$. The equilibrium distribution function for temperature field is as follows,

$$g_i^{(eq)} = w_i T\left[1 + \frac{3(\mathbf{e_i}.\mathbf{u})}{c_s^2}\right] \qquad (14)$$

The macroscopic temperature for temperature field is

$$\rho T = \sum_i g_i(\mathbf{x}, t) \qquad (15)$$

Thermal equation through Chapman-Enskog multi-scale expansion is obtained as,

$$\frac{\partial(\rho T)}{\partial t} + \nabla.(\rho u T) = \alpha\nabla^2(\rho T) \qquad (16)$$

Two-dimensional flow

Flow structure generally depends upon driving forces such as Rayleigh or Reynolds numbers. At low range of these driving forces flow is steady and stable. At Re=25, Ra=15000, and Pr=1 flow doesn't attain self sustained oscillations. It behaves as a steady streamline. Figures 1(a) and 1(b) shows steady streamlines and isotherms for the flow with periodicity n=1 and $L_x=2\pi$.

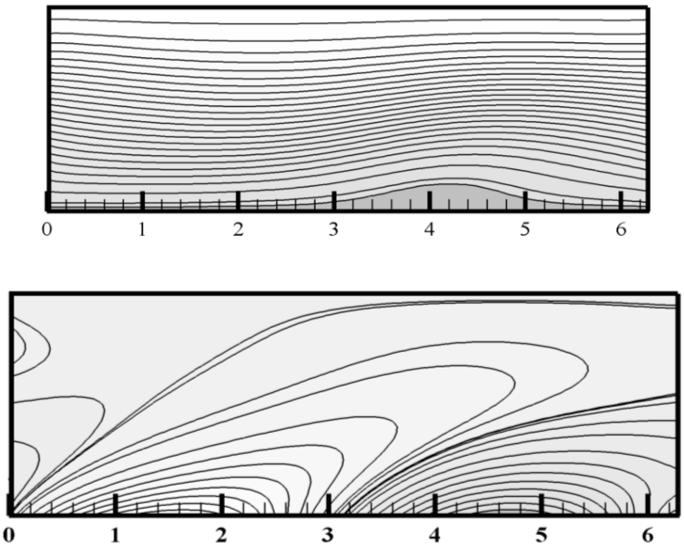
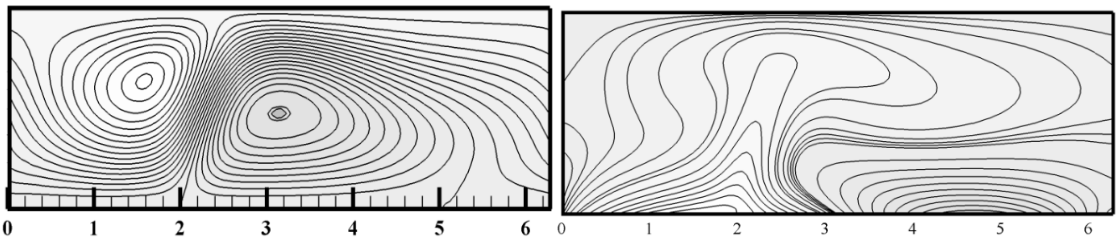


Fig 1(a) streamlines and 1(b) isotherms for Re=25, Ra=15000, Pr=1, n=1, $L_x=2\pi$

There doesn't exist any separation vortex in streamlines. Due to spatially periodic temperature at the lower wall, a plume of hot fluid is being pushed out toward the downstream position of channel. No change in vertical direction appears. This behavior of flow is due to forced convection. Buoyancy forces are not dominant in this case. These results are in good agreement with Tangborn (1992), who carried out numerical simulation of two-dimensional mixed convection flow with range of Re=20, Ra=10000.

Figure 2(a) and 2(b) shows streamlines and isotherms for Re=5, Ra=15000 with periodicity m=1 and $L_x=2\pi$.

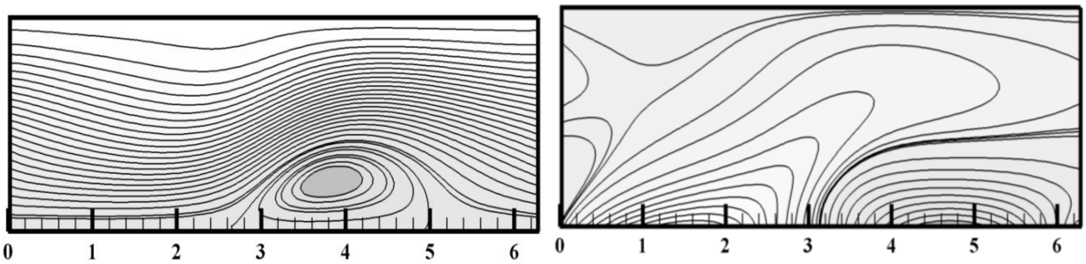


2(a) **2(b)**

Fig 2(a) streamlines and 2(b) isotherms for Re=5, Ra=15000, Pr=1, n=1, $L_x=2\pi$

In that case byouncy forces are dominant . Streamlines shows the sepration just downstream from hot section of wall as well second sepration occurs on upper surface. After a time lower wall sepration vortex increses in size then subdivides into two vortices. In fig 2(b) a plume of hot wall moves upward in vertical direction and then moves in downstream position through channel flow. This result is also similiar to Andrew V. Tangborn with Re=5, Ra=20000 and Pr=1.

At Re=15, Ra=20000, figure 3(a) and 3(b) shows streamlines and isotherms with periodicity m=1, $L_x=2\pi$. In that case flow structure develops due to the effects of both pressure and bouncy forces. The separation vortex that generated at downward position moves upward and reaches at the centre of channel. While the isotherms show horizontal as well vertical motion of hot plume generated at the bottom of channel. This flow behavior is similar to tangborn balanced flow at Re=10 and Ra=15000.In our case balanced flow that hold for an instability at $\frac{Ra}{Re^2} = 89$. When the driving forces increase, then flow turn out toward instability condition.



3(a) 3(b)

Fig 3(a) streamlines and 3(b) isotherms for Re=15, Ra=20000, Pr=1, n=1, L_x=2π

Figure 4(a)-4(g) show the instantaneous streamlines for the case Re=10, Pr=1, L_x=2π and m=1 for one cycle of oscillation with different range of Ra number. Firstly, separation vortex generated at downstream position. Then it moves to upward direction and reaches about one half of channel height. A secondary vortex with primary vortex also generated on upper surface of channel but small in size as compared to primary vortex. As Ra increases, lower wall vortex separation elongates and moves to downstream position. These two-separation vortex completely separated from each other inside channel and form an irregular pattern of flow due to bouncy effect. Then behavior of flow changes from steady to unsteady.

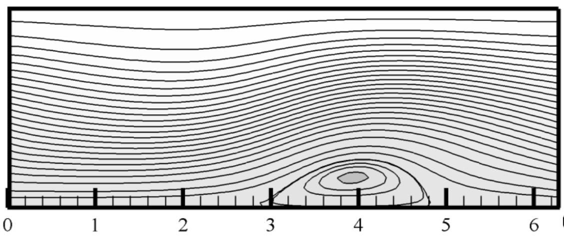


Fig. 4(a) Re=10,Ra=5000

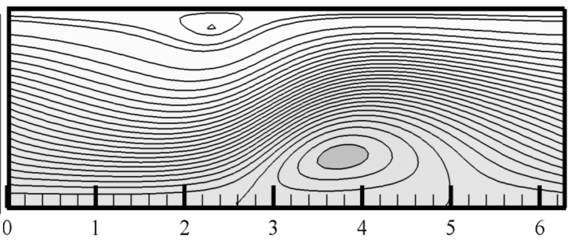


Fig. 4(b) Re=10,Ra=1000

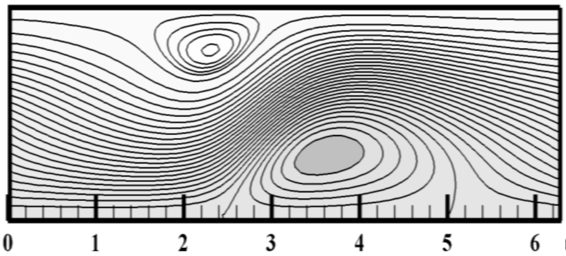


Fig. 4(c) Re=10, Ra=15000

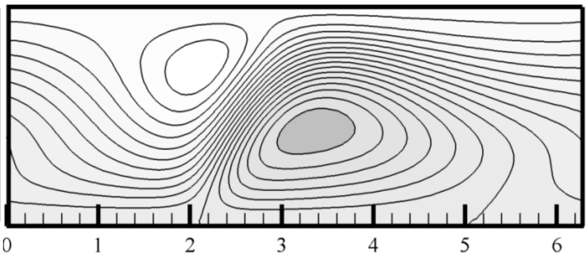


Fig. 4(d) Re=10, Ra=40000 Re=10, Ra=70000

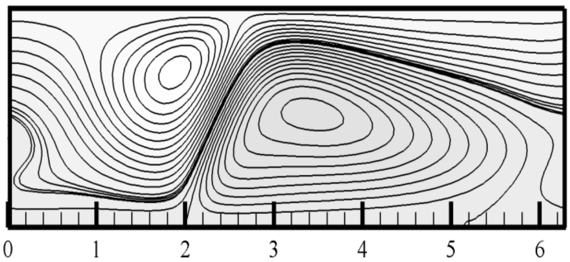


Fig. 4(e) Re=10, Ra=70000

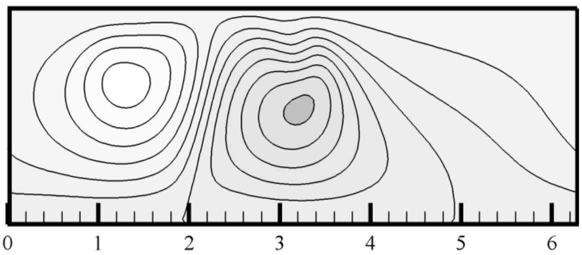


Fig. 4(f) Re=10, Ra=85000

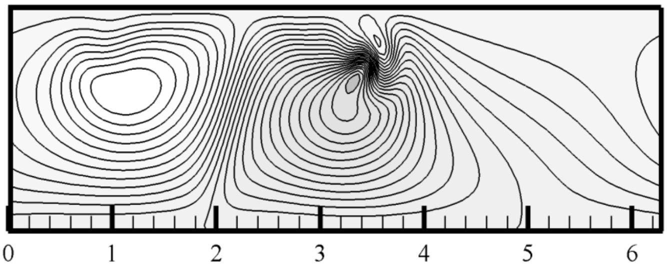


Fig. 4(g) Re=10, Ra=100000

Figures 5(a)-5(d) show instantaneous isotherms for same cycle of oscillation. A plume of hot fluid pushed downstream that has formed above the hot section of lower wall. Due to increase in Ra number, number of oscillations increase as well length of plume. Vertical motion will also occur due to bouncy and pressure forces. The plume begins to cool and drop towards the lower wall. At a certain range of driving force Ra the flow form a complicated structure. it is due to irregular behavior of flow. Now flow is no more stable and steadier. It is unstable.

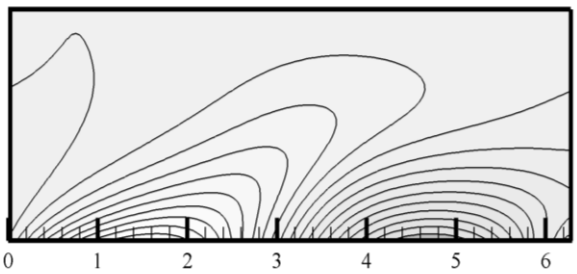


Fig. 5(a) Re=10,Ra=5000

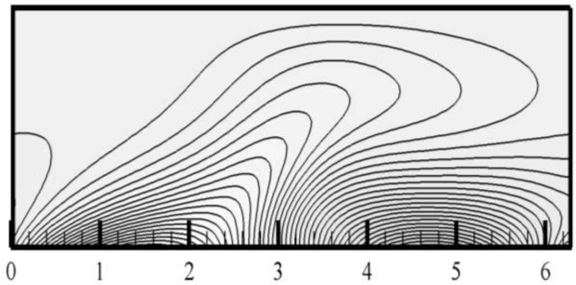


Fig. 5(b) Re=10,Ra=10000

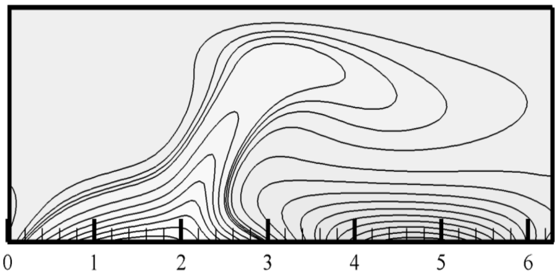


Fig. 5(c) Re=10, Ra=40000

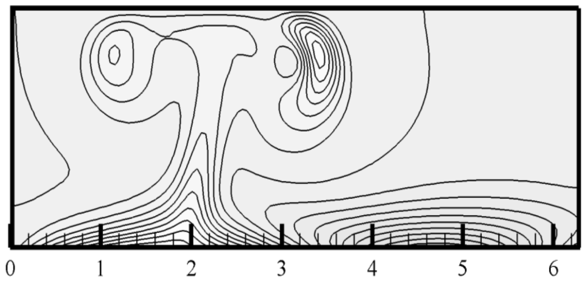


Fig. 5(d) Re=10, Ra=40000

When flow is due to Rayleigh-Benard convection. Then by increasing the parameter (Ra,Re) , simple laminar state becomes unstable to complex flow pattern through a sequence of bifurcation. Each bifurcation leads to a state that is more complex than preceding, and each state is stable over some range of control parameter. The flow becomes turbulent only after several such bifurcations.

Figure 6(a)-6(d) showing velocity time series measurement at different value of Re and Ra

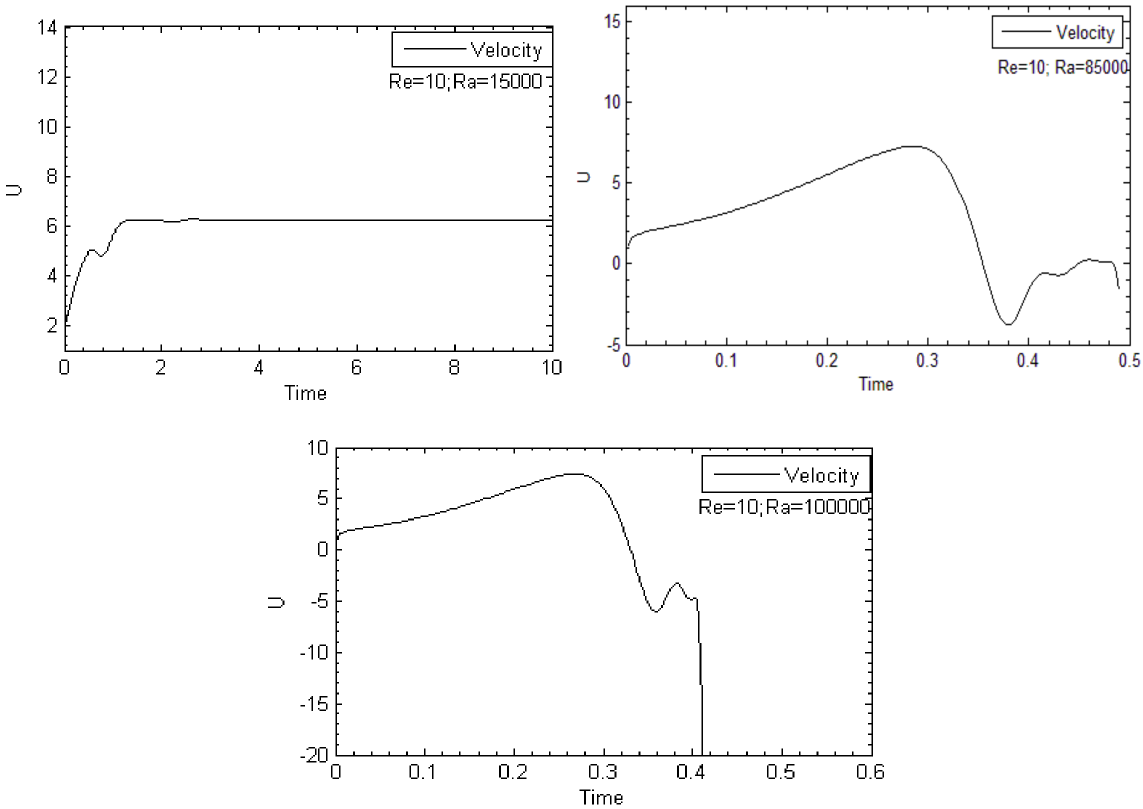


Fig. 6(a)-6(c) Velocity time series

Figure 6(a) doesn't show any rapid change in velocity with time. A small oscillation occurs at t=1. The flow behaves smoothly. Which shows steady and stable behavior of flow. Figure 6(b)and 6(c) shows rapid change in velocity due to irregular motion of flow. Its velocity representing ascending and descending behavior with time. Due to this flow is unstable as Ra increases and reaches to its certain range. When we take m=1, the boundary condition periodicity is same to box length. when we take n=2, then periodicity will be reduced. The most unstable wavelength could be longer than the periodicity of geometry and this instability cannot be visualized. Fig 7(a)-7(b) shows streamlines and isotherms for Re=10, Ra=10000, Pr=1, n=4 and $L_x=2\pi$. The flow periodicity same to boundary condition periodicity from four vortices. This result is good agreement with Tangborn (1992) having Re=10, Ra=15000. Figure 7(a) shows steady streamlines with four vortices generated at lower surface. While isotherms show four hot plumes of fluid moving in downstream position due to to spatially periodic lower wall heating.

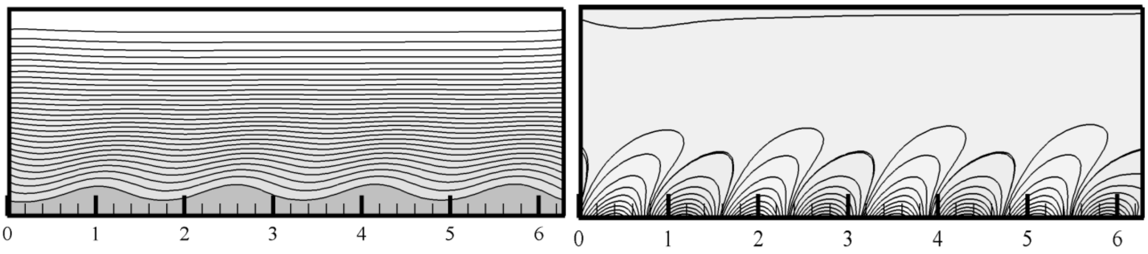


Fig 7(a) streamlines and 7(b) isotherms for Re=10, Ra=10000, Pr=1, n=4, L_x=2π

Figure 8 shows time history of velocity for case Re=10, Ra=10000, Pr=1. When Re increased then flow contains self sustain oscillation. In that case decay in velocity is quite rapid till t=2.5, then velocity remains constant till t=20.

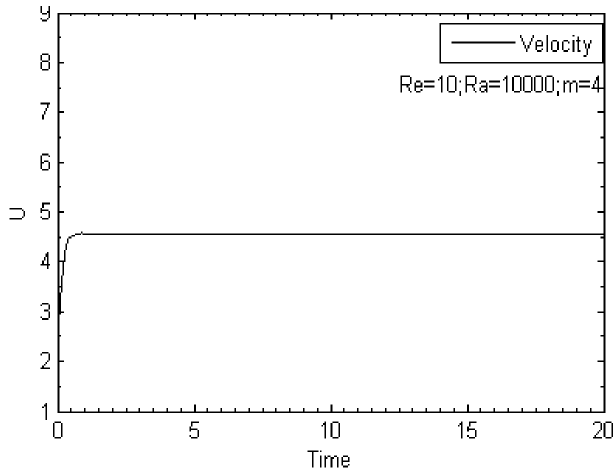


Fig. 8 Velocity time series

According to Wang et al. (1991) and Tangborn (1992) the steady and unsteady behavior of flow depends upon certain values of Re and Ra. Fig 9 shows stability behaviour of flow for different values of Re and Ra. It is clear from figure that for small values of Re flow turn out to be unsteady earlier for different Ra numbers as compared to large values of Re and Re. As we increase Re, it takes time for flow to decay a steady state. If byouncy forces are dominant then flow becomes unsteady sooner.

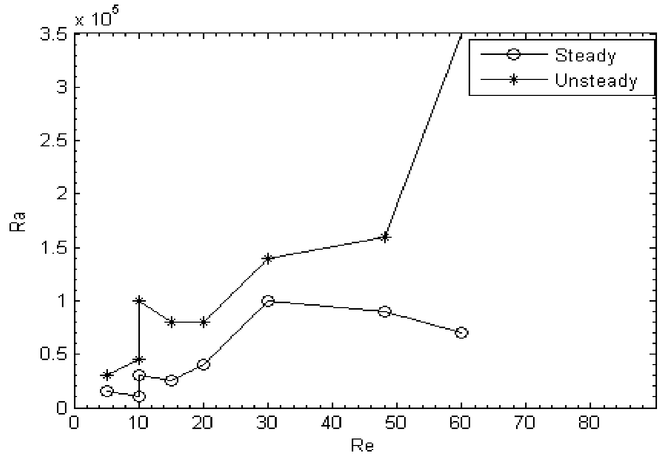


Fig. 9. Stability diagram for L_x=2π, n=1, Pr=1

References

1. J.M. Luijkx, J.K. Platten, L.Cl. Legros, On the existence of thermo convective rolls, transverse to a superimposed mean poiseulle flow, Int. J. Heat Mass Transfer 24 (1981) 1287-1291.
2. K.C. Chiu, F. Rosenberg, Mixed convection between horizontal plates, I: entrance effects, Int. J. Heat Mass Transfer 30 (1987) 1645-1654.[3] K.C. Chiu, J. Ouazzani, F. Rosenberg, Mixed convection between horizontal plates-II. Fully developed flow, Int. J. Heat Mass Transfer 30 (1987) 1655-1662.
3. G.H. Evans, R. Grief, A study of traveling wave instabilities in a horizontal channel flow with applications to chemical vapor deposition, Int. J. Heat Mass Transfer 32 (5) (1989) 895-911.
4. M.T. Ouazzani, J. K. Platten, A. Mojtabi, Etude experimentale de la convection mixte entre deux plans horizontaux a temperatures differentes-II, Int. J. Heat Mass Transfer 33 (1990) 1417-1427.
5. Ostrach and Y.Kamotani, " Heat transfer augmentation in laminar fully developed channel flow by means of heating from below," J. Heat Transfer 97, 220 (1975).
6. D. G. Osborne and F. P. Incropera, " Laminar mixed convection heat transfer for flow between horizontal parallel plates with asymmetric heating, " Int. J. Heat Transfer 28, 207, (1985).
7. S. Orszag, " Accurate solution of the Orr-Sommer field stability equation," J. Fluid Mech. 50, 689 (1971).
8. F. S. Michael, Dwight Barkley and Harry L. Swinney," Instability in a spatially periodic open flow," phys. Fluids 7(2), February 1995.
9. channels. Part 1. Stability and self-sustained oscillations," J. Fluid Mech. 163, 99 (1986 a).
10. N. K. Ghaddar, M. Magen, B. B. Mikic, and A. T. Patera, "Numerical investigation of incompressible flow in grooved channels. Part 2. Resonance and oscillatory heat-transfer enhancement," J. Fluid Mech. 168, 541(1986 b).
11. K. J. Kennedy and A. Zebib, " Combined free and forced convection between horizontal parallel plates: Some case studies, " Int. J. Heat Mass Transfer 26, 471 (1983).
12. X. Wang, L. Robillard, and P. Vasseur, " Laminar mixed convection heat transfer between parallel plates with periodically localized heat sources, " Heat Transfer Develop. 171, 81 (1991).

Tangborn, "A two dimensional instability in a mixed convection flow with spatially periodic temperature boundary conditions, " Phy. Fluids A 4, 1583 (1992).