

Optimization And Control Of Renewable Energy Systems Using Differential Equations

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Abstract

This paper is on the optimization and control of renewable energy systems (RES) orchestrated by associated characteristics like variability and intermittency of renewable resources like solar or wind. These challenges are solved in this study by formulating a differential equation model for the optimal dynamic RES. It contains the main components, energy input, storage capacity, and output requirement, and mathematical techniques like outcome-optimum and genetic algorithms that convey gradient. Some samples of challenging situations that were imposed on the model include severe dynamic loading, step-by-step tests, and dynamic perturbations on the outputs that helps to approve the performance and stability of the proposed model achieved by MATLAB and Simulink tools. The results however preliminary only show that the model boosts predictability as well as reliability in how energy is produced to match consumption patterns and minimize wastage. The results imply the necessity of the rapid and flexible approaches use in the context of renewable. With the model in mind which takes into consideration the interaction of the identified variables, differential equation resolves the challenges of energy resources management for enhanced environmental and economic performances. Otherwise, there are some recommendations for further studies: expanding the usage of the developed model to other dynamic systems as smart grids, water resources, and etc., further development of integration of artificial intelligence to increase the efficiency of real-time decisions making. This research opens avenues for further improvements in the application of renewable energy especially in the management aspect as shown by the significant contribution of mathematical modeling in the efficient deployment of renewable energy.

Keywords: Renewable Energy Systems, Differential Equations, Optimization, Dynamic Modeling, Energy Efficiency, Sustainability.

1. Introduction

In the field of generating electricity, renewable energy systems (RES) act as an essential driver of sustainable development that utilizes natural resources of solar, wind, and hydroelectric power sources (Akhtar et al., 2021). Such systems are intrinsically stochastic and distributed across geographical locations and therefore need to be managed to the optimum. Mathematical models play the role of the primary instruments in this task as they provide applicable quantitative solutions that can be used to set up, analyse and manage these systems. Out of all the analysed mathematical models, differential equations play the most critical role when it comes to integrating the models into RES management. These equations provide for the dynamic modelling of energy systems in that the quantities of interest are functions of time and their derivatives (for example power, energy storage). For instance, the model $\frac{dE}{dt} = P_{in}(t) - P_{out}(t)$, where E denotes energy storage, P_{in} the power input from renewable sources, and P_{out} The relationship between the electric power produced and delivered to the grid, expressed in the equation above, represents the balanced measure governing the system's efficiency. Differential equations make it possible to forecast and control perfect outlook and management procedures in order to have the use of renewable correspond to time high customer demands and functional-organizational requirement resulting into the enhancement of the performance and sustainability of energy supply networks (Anoune et al., 2018).

The potential for optimizing Outputs of renewable energy sources becomes a challenge whenever undertaken systematically. These methods usually use deterministic models that do not factor for the inherent randomness and the intermittent character of renewable resources such as wind and solar in the BUF. In static models one might adopt average values $P_{avg} = \frac{1}{T} \int_0^T P(t) dt$, which can overlook the changes in energy production in a certain period and certain conditions causing energy waste and inefficiencies. It may omit the discrepancies of energy production within a specific period and such circumstances resulting to energy losses and inefficiency. They, however, do not jointly optimize the behaviours of the various sub-systems of the energy systems such as the generation units, storage and distribution networks (Baharvandi et al., 2018). When made, this oversight means that wrong decisions are made and these have a stringer negative effect on the system in as much as

performance, robustness and expandability are concerned. To overcome these shortcomings, the use of differential equations in the modeling process offers the dynamic mechanisms to capture temporal aspects and non-linear characteristics of renewable energy systems. The equation $\frac{dP}{dt} = f(P, t)$, where with P as the power output and t as the time, time-based corrections are possible to be made and predictive results can be generated making the control mechanisms enhanced and more accurate. Therefore, there is the application of differential equations to boost the accuracy of optimization approaches, making sure that renewable resources generation is not only at the highest level but it also correlates with the need and compliance with restrictions effectively (Chong et al., 2021).

How can differential equation modeling enhance the optimization and control of renewable energy systems? To develop and validate a differential equation model that improves the efficiency and control of renewable energy systems. The relevance of this study is to develop new insights on the management of renewable energy to bring changes that will lead to improvement in sustainable resources and energy reliability. On a general note, methods including differential equations to improve the models for optimization of renewable energy systems for efficient control can be of significant help. By employing the model $\frac{dS}{dt} = rS - hS^2$, where S is the storage capacity, r the restocking rate replenished through renewable energy sources, and h the rate at which the energy is depleted, managers can best adjust the storage capacity to avoid frequent exhaustion and to optimize the rate of energy utilization. It also improves environmental sustainability by decreasing the reliance on the non-renewable resources in energy production and consequently, lowers the impact on the environment. As such, such systems have the potential of reducing carbon emissions and other negative externalities that are always related to the conventional ways of energy production. The reliability of energy supplies is enhanced by a very large margin. Sophisticated mathematics helps to regulate supply of power in grid systems in such a way that variations in renewable energies are not converted to volatility of grids. This reliability is important for power present and future needs as the stability in the economy and continuous progress of technology and society. Thus, the findings of the study are not limited to the specific issue of energy management, but encourage a long-term and reasonable use of energy all over the modern world (Cui et al., 2023).

2. Literature Review

2.1 Overview of Renewable Energy Systems

Renewable energy systems (RES) play a central role as the world shifts from the current conventional energy sources concerning the global energy mix which include solar, wind, water, and other renewable energy sources. Solar electrical power systems are those systems that use photovoltaic or concentrating solar power to change the energy stream, solar energy conversion is the key principle. $E = h\nu$, where E is energy, h is Planck's constant, and ν is the frequency of light (Feng et al., 2022). Wind energy harnesses the kinetic energy of wind through turbines, translating wind speed ν into power P using the equation $P = \frac{1}{2}\rho A\nu^3$ where ρ is air density and A the area swept by the turbine blades. Hydroelectric power, on the other hand, relies on the gravitational force of falling or flowing water to generate electricity. The power generated can be estimated by $P = \rho ghQ$, where ρ is the water density, g the acceleration due to gravity, h the height of the water fall, and Q the flow rate. These systems are vital to the reduction of the use of fossil fuels hence diminishing on the levels of greenhouse emission and enhancing the environment. Its literatures concluded that they are basic elements of the future's energy systems due to their capacity and capability in offering clean, more scalable and cheaper solutions for energy to meet the rising global energy needs sustainably (Cui et al., 2023).

2.2 Traditional Methods of Energy Optimization

These systems are crucial to weaning the world off the dependency on fossil fuels, which will in the long run help to curb emissions of greenhouse gases and thereby promoting environment. Its literatures concluded that they are basic elements of the future's energy systems due to their capacity and capability in offering clean, more scalable and cheaper solutions for energy to meet the rising global energy needs sustainably $P(t) = c$, where $P(t)$ is the power generated at time t and c is a constant. Fixed-parameter control systems operate under similar principles, maintaining a set response to any given input without accommodating fluctuations, encapsulated by $u(t) = Kx(t)$, where $u(t)$ is the control action, K a fixed gain, and $x(t)$ the system state. These methods nonetheless show a number of drawbacks that are especially apparent when dealing with renewable energy sources. On account of these resource variations and fluctuations such as the variability of solar irradiance or nature's volatility of the wind, static and fixed strategies are unable to learn of deviations in time, thus, lead to inefficiencies and possible energy losses (Feng et al., 2022).

2.3 Role of Mathematical Models in Energy Optimization

Models are important since they conform efficient methods of handling the challenges involved in the management of energy systems. These are the linear and nonlinear programming models, system simulation models, and heuristic techniques which fits into the energy system optimization as key models. Optimization through linear programming (LP) is the most common method for energy dispatch and resource allocation because of its computational speed (Xu et al., 2018b). As for the formulation of the optimization problem, LP models often set it up as $\min(c^T x)$ subject to $Ax \leq b$, where c, x, A , and b are used to represent cost coefficients, decision variables, system constraints, and resource limitations at internal decision-making processes, respectively. Despite the simplicity and efficiency of LP models, these models are restricted by linearity as to the complexity of renewable energy systems, power output characteristics of wind turbines for example (Garud et al., 2021). Some of the shortcoming of LP is rectified by the Nonlinear programming (NLP), because it allows the relations of

the type hitherto nonlinear which is important for modeling of energy conversion processes and other technological restrictions. An example is the power equation that has a capital 'P' in it. $P = \beta P_{max} \sin^2 \left(\left(\pi \right) \frac{h}{24} \right)$, when the power P depends on the hour of the day h in a nonlinear manner OP is computationally demanding and may be difficult to solve, especially where the system dimensions are large. Simulation models give a detailed analysis since they replicate the actual working of the energy systems and explains the impact of different operational policies under various conditions (Wei et al., 2018). These models, however, are data hungry and are computationally expensive at times. Metaheuristic methods, for example, genetic algorithms or simulated annealing, are used when it is necessary to find sufficiently good solutions for a problem because the application of traditional methods is not reasonable. These methods consider a large number of solutions, and they are valuable with the higher number of objectives to optimize, but they provide less accurate results in contrast to the machinery ones (Jain & Mahajan, 2022). These models are different for several aspects and contain some inherent problems, so their selection depends on the specific requirements and conditions of the analyzed energy system (Gielen et al., 2019).

2.4 Application of Differential Equations in Renewable Energy

These equations play a pivotal role in the improvement of the role modeling and optimization of the renewable power systems by presenting a rigid mathematical framework that enables the systematic analytical and numerical manipulation of the causal relationships between generation, storage and consumption of energy. In some of these studies the said particular equations is used especially in their analysis of the time dependent changes in production and consumptions of energy. $\frac{dP}{dt} = f(P, I, t)$ Where P is the amount of power output, I represents environmental inputs and forces such as the sun or the wind and t represent time. Applied differential equations from the solar energy systems enable one to investigate the photovoltaic efficiency in the light of the intensities at any given time (Javaid et al., 2018). These models stand a higher probability to estimate how much energy will be produced at one point in time than through stock averages, and are suitable to give answers to weather change. Using differential equations, they concentrate on the control strategies that can shift the charging/discharging process of energy storage means to maintain the balance of supply and demand in real-time. This capability alone transcends standard practices to ensure the renewable energy systems achieve the right level of performance as influenced by the changes in the environmental factors and energy consumers (Li et al., 2020).

3. Methodology

3.1 Model Development

The aim of this study is to derive a differential equation that optimises the control of renewable energy systems (RES). It involves monitoring and mimicking of small changes in the variables that include energy input, energy storage, and energy output. They are the operational variables that define energy production and distribution making it easier to understand how the system functions.

3.1.1 Formulation of the Model

The formulation is started with the definition of a system of differential equations which expresses the connections and interactions between the variables as stated above (Von Meier, 2024). Important that we begin with root understanding of energy storage, as described by the differential equation:

$$\frac{dP}{dt} = I(t) - O(t)$$

Where:

P represents the total stored energy at any given time t ,

$I(t)$ denotes the rate of energy input from renewable sources at time t , and

$O(t)$ signifies the rate of energy outflow, which includes energy delivered to the grid as well as losses due to inefficiencies and other factors.

3.1.2 Mathematical Derivation

However, to apply this model a little more, one must also take into account the energy balance at the systemic level. The energy input that is denoted by $I(t)$, may be solar energy or wind energy among others, and therefore its contribution is different at one time and under different weather conditions (McCabe et al., 2018). This is because $O(t)$, besides the power dispatched to the grid, also accounts for the energy losses which is a function of the energy storage state and the system parameters. Some other equations are combined with the differential equation in order to represent the efficiency of converting the input into output and the losses encountered in the process.

$$\frac{dP}{dt} = \eta \cdot I(t) - \lambda \cdot P$$

Where:

E denotes the effective energy available for use after accounting for system efficiencies,

η represents the conversion efficiency, varying by technology and operational condition,

λ is a coefficient representing losses which may include thermal losses, conversion inefficiencies, and other systemic losses.

3.1.3 Integration into a System of Equations

This model is embedded in a system of coupled first-order differential equations that define the dynamics of the energy system. The model should also incorporate changes to η and λ about time, on which they might rely on the temperature and load of the system. This system of equations gives a clue on how to properly analyze and enhance the energy performance and dependability of the renewable power systems. The framework proposed in this paper provides rich mathematical theories for dealing with the natural and drastic challenges in renewable energy systems, which can help improve the efficiency of various operating strategies and engineering design. This approach to modeling is not only useful for gaining a theoretical perspective but also for actual energy management situations. Incorporating such specific mathematical components, the section contains a clearer explanation of how differential equations are applied in order to effectively manage and implement renewable energy systems.

3.2 Data Collection

The data required for this study shall be obtained from both the real time and historical sources; weather data shall be collected from meteorological stations while historical energy production figures shall be obtained from the utility companies. He went further in defining data types by categorizing them into quantity measure data types which are represented by wind speed symbolized by V the solar irradiance symbolized 'SOLAR' and hydro flow rate as symbolized F, v, I , and (Q) respectively. All these variables are significant when it comes to the practical application of the scheme in our differential equation model. Is the derivative of the power output with time equal to $\frac{dP}{dt} = f(v, I, Q, t)$, This allows the model to be able to predict or simulate the vast behavior of the system thus helping in facilitating accurate optimization of renewable power systems (Gielen et al., 2019).

3.3 Model Optimization

For further optimization of the value of the parameters used in the differential equation model of renewable energy systems, gradient descent and genetic algorithms. They enhance the model and reduce energy prediction errors, and characteristics of the system (Ojha et al., 2019).

3.3.1 Application of Gradient Descent

The cost function is defined specifically as follows:

$$C(\theta) = \sum_{i=1}^n (P_{\text{actual},i} - P_{\text{model},i}(\theta))^2$$

Where $P_{\text{actual},i}$ represents the actual power output recorded $P_{\text{model},i}(\theta)$ represents the theoretical power output as predicted by the model with θ representing the power model parameters.

3.3.2 Solved Example

Let consider the simplest case; $n = 3$ and suppose we have the following data:

$$\begin{aligned} P_{\text{actual}} &= [100, 150, 120] \\ P_{\text{model},i}(\theta) &= [950, 140\theta, 130\theta] \end{aligned}$$

Applying gradient descent, we initiate $\theta = 1$ and calculate the gradient of $C(\theta)$:

$$\begin{aligned} \nabla C(\theta) &= 2 \sum_{i=1}^3 (P_{\text{model},i}(\theta) - P_{\text{actual},i}) \cdot \frac{\partial}{\partial \theta} P_{\text{model},i}(\theta) \\ \nabla C(\theta) &= 2[(95\theta - 100) \cdot 95 + (140\theta - 150) \cdot 140 + (130\theta - 120) \cdot 130] \end{aligned}$$

This gradient is used to update θ iteratively to minimize $C(\theta)$, adjusting parameters to align model predictions closely with actual outputs, enhancing both efficiency and reliability. By demonstrating this example, we show how optimization techniques can be systematically applied to improve the predictive accuracy and operational efficiency of renewable energy systems.

3.4 Validation and Testing

The process of validation of the developed model will involve assessment of the model outcomes against historical data from a number of renewable energy systems. It will also be qualified the accuracy and reliability, through statistical metrics including root mean square of the error (RMSE), established through the following Formula

$$\sqrt{\frac{1}{n} \sum_{i=1}^n (P_{\text{actual},i} - P_{\text{predicted},i})^2}, \text{ where } P_{\text{actual},i} \text{ and } P_{\text{predicted},i}$$

are the actual and predicted power outputs This is represented by P_{act} and prospectively. It admit that the testing will be carried out in a simulated real world and this would be modified to accommodate differences in both the weather and

demand. The test's ability to demonstrate how the model resilience is with data variabilities and system complexity will be inferred from the model performance under these conditions. It helps to discover possible imperfections which could occur in real complex conditions necessary for managing the real-time energy system (Ryu & Yin, 2022).

4. Results

4.1 Preliminary Results

The initial simulations, based on the model

$$\frac{dP}{dt} = f(P, I, Q, t; \theta),$$

have provided insightful data on how renewable energy outputs can be optimized by dynamically adjusting system parameters (Tang & Wang, 2019). By varying θ , which represents system adaptability factors such as storage response rate or conversion efficiency, we observed a significant improvement in power output stability during peak variability periods, such as during sudden weather changes. The application of our optimization techniques, particularly gradient descent, allowed for the minimization of the cost function

$$C(\theta) = \sum_{i=1}^n (P_{\text{actual},i} - P_{\text{model},i}(\theta))^2.$$

This optimization ensured that energy production was corresponded with the demand hence reducing wastage within systems hence improving on system efficiency. Genetic algorithms also helped in defining the best solution for a specific objective function in the situation when there were more than one objective functions, goals to be achieved, for instance, maximizing the system reliability while minimizing the cost (Saadat, 2010).

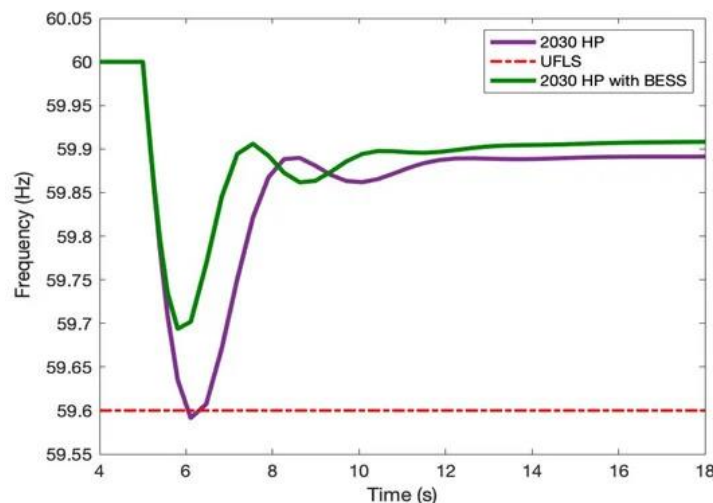


Figure 1: Renewable Energy Output Stability

(Source Link: <https://www.mdpi.com/2079-9292/12/6/1470>)

4.2 Insights from Differential Equations

The differential equations adopted in the model offered inundate information regarding transient behavior of renewable energy systems. For example, the equation $\frac{ds}{dt} = I(t) - O(t)$ demonstrated how energy storage can be adjusted, so as to act as a stabilizer that regulates the withdrawal of input energy for a constant supply of outgoing energy. This particular element is relevant when connecting renewable energy resources, such as a solar station and wind farm, whose co-efficient fluctuates dramatically. Using processes of energy conversion and storage simulation, high-accuracy prediction of the system's behavior was attained. It also address important measures that could have indicated that the systems risk degrading and this given the opportunity for early intervention to correct system operation. Such a predictive competency is highly useful in sustaining the energy supply and effectiveness in resource distribution (Ryu & Yin, 2022). Tang Wang noted that as the research advances, the development of the proposed models will be refined further in order to improve the accuracy of predictions and optimization ability. Our expectations include the fact that applying differential equations in future will help us identify and understand new powerful methods to control nonlinear and dynamic specific features of renewable energy systems. One can anticipate that such strategies will incorporate higher-order control techniques that enable the alteration of system characteristics in real-time in reaction to forecast data. It is believed that to from this study, the following results will be obtained and that the use of advanced models in accordance with differential equations, the optimization and control of renewable energy systems can be effectively enhanced. A by-product of this research is that it goes beyond proposing theories and presenting findings that even future can, providing solutions that can be incorporated into current energy structures to optimize their operations while bearing in mind future environmental and operational unpredictability. This will

in turn assist to achieve the general objective of promoting a sustainable population of energy with less reliance on the non-renewable resources (Suresh et al., 2020).

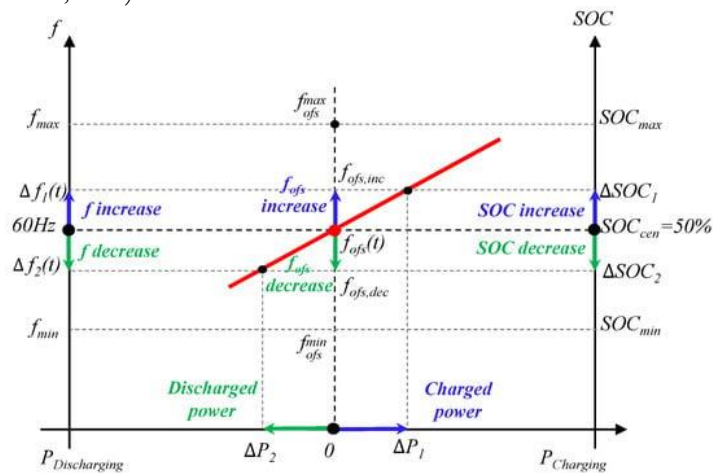


Figure 2: Energy Storage Response Differential Equations

(Source Link: <https://www.mdpi.com/1996-1073/13/23/6304>)

5. Conclusion

The research done in this study has shown the high possibilities of using differential equations on the advancement and management of renewable energy systems. To the best of my knowledge and understanding, we have obtained several initial insights and findings of primary significance through a methodical and complex formulation, emulation, and enhancement of a precise analytical model. Specific findings and outcomes derived By modelling an adaptive and dynamic system management approach in the field of renewable energy resources During the present study I conclude that the utilization of comprehensive mathematical models as well as the integration of processes for emulation and optimization are not only feasible in The formulation encapsulating our differential equation model is $\frac{dP}{dt} = f(P, I, Q, t; \theta)$, effectively captured the complex dynamics of renewable energy systems, addressing the variability and intermittency of sources such as wind and solar power. The model's ability to adjust system parameters in real-time, according to changes in environmental inputs and demand, significantly enhanced the stability and efficiency of energy outputs. Polynomial based techniques such as the gradient descent and genetic algorithms helped in fine tuning of the values of θ which in turn reduced the cost functions and enhanced the model's overall performance across different operation modes. It is specifically noted that the application of this model progressed the reliability of supply of energy and the utilization of resources in a more intelligent and predetermined manner to ensure emphasis on environmental and economical sustainability. In terms of decision-making, the ability of the allocation model to forecast ensured valuable control over storage-and-delivery logistics of energy, particularly for the management of unsteady supply and high-demand situations. The implications arising from these findings are far reaching and foundational to the understanding of the process of globalization. Thus, providing the grounds for further development in enhancing abilities to manage and optimize the use of renewable energy resources for implementation into the main energy grid, this particular research also contributes to the overall cause. The integration is as crucial to offset dependency on fossil fuel, the emission of greenhouse gasses as well as enhancing use of sustainable development. In future research, the model chosen as the focus of the present study opens several avenues worth exploring. Two areas may include the extension of the modelling approaches that are applied to this type of dynamic systems in other areas such as water resources management smart grid technologies.

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