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Advanced Analysis of Heat and Mass Transfer in MHD Flow Over a Stretching Sheet with Radiative Effects Using the Shooting Method

Sadia Irshad[1*], Shah Jahan[1], Eduardo García Villena[2,3,4], Henry Fabian Gongora[2,5,6], Muhammad Hussain Ali[1]

[1*]Institute of Mathematics, Khawaja Fareed University of Engineering and Information Technology, Rahim Yar Khan, Punjab 64200, Pakistan. Sadiairshad15@gmail.com, jahanshah@kfueit.edu.pk, alidkd4432@gmail.com,
[2]Universidad Europea del Atlántico. Isabel Torres 2139011 Santander, Spain. henry.gongora@unini.edu
[3]Universidad Internacional Iberoamericana Arecibo, Puerto Rico 00613, USA. eduardo.garcia@uneatlantico.es
[4]Universidade Internacional do Cuanza. Cuito, Bié, Angola.
[5]Universidad Internacional Iberoamericana Campeche 24560, México.
[6]Universidad de La Romana. La Romana, República Dominicana, mx

***Correspondence Author:** Sadia Irshad
*Email: sadiairshad15@gmail.com ;jahanshah@kfueit.edu.pk; eduardo.garcia@uneatlantico.es

Abstract:
Purpose- The main focus of this investigation is to study heat and mass transfer dynamics in the field of magnetohydrodynamic (MHD) boundary layer flow using the shooting method. Understanding these mechanisms is important for optimizing MHD systems in various engineering domains such as materials processing, aerospace, and energy.
Design/Approach/Methodology- The study employs the shooting technique, a powerful numerical tool, to solve complex equations related to heat transfer and fluid dynamics in MHD flows. Through this method, the research seeks to understand velocity, temperature, and concentration profiles. The study also uses parametric analysis to explore the complex relationships affecting mass and heat transfer rates, considering variations in Prandtl number, magnetic field intensity, and Schmidt number.
Findings- The numerical solutions provide insights into velocity, temperature, and concentration profiles within the MHD boundary layer flow. The parametric analysis reveals complex linkages that influence mass and heat transfer rates. Factors such as Prandtl number, magnetic field intensity, and Schmidt number significantly affect these rates.
Originality/value- The research highlights the combined effects of fluid behavior, magnetic effects, and thermal-mass exchange. These insights will assist designers in creating more sustainable and effective MHD systems. The findings are valuable for enhancing the efficiency and sustainability of MHD applications in various engineering fields.

Keywords MHD, Heat and Mass Transfer, Stretching Sheet, Shooting Method, Radiation.

Paper type Research paper

1. Introduction

Heat and mass transfer within magnetohydrodynamics (MHD) boundary layers play a critical role in enhancing energy efficiency across aerospace, industrial processes, and renewable energy systems. These transfers are impacted by the way magnetic fields and electrically conductive fluids interact, significantly impact material processing, environmental factors, and the performance of renewable energy technologies. The shooting method, a fundamental numerical technique, assists in addressing the complexity of MHD boundary layer problems, enabling precise modeling for engineering designs and optimizations. A thorough understanding of these phenomena is essential for driving technological advancements and fostering sustainable practices across diverse applications.

The viscous flow of MHD that contributes to the mechanism of extrusion increased financial effectiveness in the current setting was examined by Khan et al., [1]. Divya et al., [2] investigated the effects of an irregular viscous fluid movement on a heat sink/source. The impact of an electric field on the thermal flow of a viscous fluid in the occurrence of disk rotation was examined by Mabood et al., [3]. Ahmed et al., [4] inspected the examination of heat transfer in viscous fluid mixed convection flow throughout a nonlinear changing surface while undergoing a magnetic field. Wang et al., [5] demonstrated heat transmission of 3D viscous fluids across an exponential surface. Hayat et al., [6] explored the laminar stream of a viscous fluid in a porous media containing nanoparticles. Krishna et al., [7] inspected the influence of ramping wall velocity and temperature, as well as an isothermal plate, on the infinite vertical flat pore surface radiative MHD rotating circulation of a viscoelastic incompressible Jeffrey fluid.

Heat and mass transfer are required for several useful uses, like cooking, heating, and cooling, to ensure efficient energy use and product quality in everyday life. Kumar et al., [8] investigated mass and heat transmission in MHD convection flow in combination across a moving pore plate that is inclined. Heat and mass transfer in MHD Casson nanofluid circulation across a stretched sheet using thermophoresis and Brownian movement were studied by Kanchi et al., [9]. Shanmugapriya al., [10] solved the heat and mass transfer-related MHD flow through the boundary layer of Casson liquid on a moving wedge. Influence of chemical reaction and twofold stratification on mass and heat transference properties of nanoliquid circulation with heat radiation across a porous stretched sheet examined by Tawade et al., [11]. Leng et al., [12] examined the application of mass and heat transfer utilizing the caputo-fabrizio derivative approach to the convective movement of Casson liquid in a microchannel. By utilizing scaling group transformations, Pavithra et al., [13] focused their investigation on magnetohydrodynamic mass and heat transfer flow over a barrier layer on a nonlinear extended surface in the presence of open convection. Agarwal et al., [14] provided exact and logical approaches that addressed the concerns of mass transport and heat transfer for MHD slip movement in nanoliquid.

Studying the behavior of magnetic fields in conductive fluids is known as magnetohydrodynamics, which has an influence on space weather prediction and fusion research on energy. Ramesh et al., [15] assessed the effects of chemical reactions on the MHD circulation of the Casson liquid utilizing a porous stretched sheet. In a two-dimension stagnation point, Mahabaleshwar et al., [16] discusses the impact of MHD and radiation on a flock traveling toward a longer surface. Cross-diffusion effects and an aligned magnetic field impact in theoretical magnetohydrodynamic (MHD) calculations were investigated by Koka and Ganjikunta [17] along a stretched surface with different thicknesses. Dissipative flow in a MHD flow pattern across a stretched sheet was inspected by Leelavathi et al., [18]. Tlili et al., [19] explored MHD stagnation Maxwell movement of nanofluid toward a stretched sheet using Brownian motion and the thermophoresis effect. Slip effects cause a reduction in boundary layer thickness and momentum transmission, as Zhou et al., [20] found when they used MHD nanofluid over a spinning surface. The MHD combined nanofluid with radiative effects and velocity slip opposes and supports the flow of the void-spaced movable surface generating heat was discussed by Sharma et al., [21].

Applications for thermal radiation are numerous in both science and technology. It is essential to numerous fields, including space research, gas engine, combustion applications, high-temperature operations, and power generation. The impacts of the MHD mixture nanofluid non-uniform flux of heat and thermal radiation were investigated in a study by Paul et al., [22]. Applying the KVL framework, Sarkar et al., [23] inspected powdered nanotube carbon nanofluid, considering the nonlinear thermal radiation's heat component. The aggregation of nanoparticles on radiative 3D Maxwell fluid flow across a permeable stretched surface with heat radiative and a heat source/sink has been numerically explored by Mishra et al., [24]. Baag et al., [25] studied the influence of viscous dissipation and radiation on the flow characteristics of magnetohydrodynamic (MHD) systems. Varatharaj and Tamizharasi [26] observed the unsteady motion of a Maxwell nanoliquid across a stretched surface while considering magnetohydrodynamic and thermal radiation effects. Seethamahalakshmi et al., [27] investigated the impact of heat radiations on non-Newtonian circulation of fluid across a stretchy surface using various factors. The effects of Joule heating and nonlinear radiation from the heat on the mobility of a magneto Casson small fluid on a sloped surface that is both porous and able to stretch was investigated by Sushma et al., [28].

The shooting technique is an essential tool for numerical analysis because it allows differential equations to be solved and provides precision and efficiency for a wide variety of applications in science and engineering. The MHD viscous fluid solution subjected to partial slip circumstances was determined by Sajid et al., [29] using both the shooting method and the homotopical analysis approach. The purpose of studying the Maxwell liquid induced convective movement via beneath the particles thermophoretic motion, Uddin et al., [30] used the RK-4 with shooting approach. Rashidi et al., [31] used the shooting technique to study MHD flow with source/sink effects on an inclination surface. Ramzan et al., [32] used the RK-method of fourth order shooting to numerically analyze magneto-nanofluid stagnation points with buoyancy force and convection boundary conditions. The numerical solution of the magnetohydrodynamic flow through the boundary layer of electrically conductive convective nanofluids under the effect of thermal radiation, viscous dissipation, and Ohmic heating, generated by a nonlinear upward stretching/shrinking sheet was obtained by Prasannakumara et al., [33] utilizing the fifth-order Runge–Kutta–Fehlberg technique with shooting method. Khan et al., [34] first studied the movement of nanofluids across a stretched surface. They also used similarity transformation to decrease unstable Navier-Stokes equations, which were subsequently estimated using multiple shot techniques. Zhu et al., [35] used the Rung-Kutta process and shooting technique to investigate the commencement of nanofluid flow on a decreasing surface that is convectively heated.

The originality of this work is evident in its unique presentation of the interconnected concepts of heat and mass transfer within magnetohydrodynamics (MHD) boundary layers and the shooting method. Through the seamless integration of these ideas, the paragraph provides a novel perspective on their profound significance across diverse engineering disciplines. It emphasizes the crucial understanding of these phenomena in advancing technology and promoting sustainable practices. This distinct approach offers valuable insights into the complexities involved in addressing MHD boundary layer challenges in engineering design and optimization.

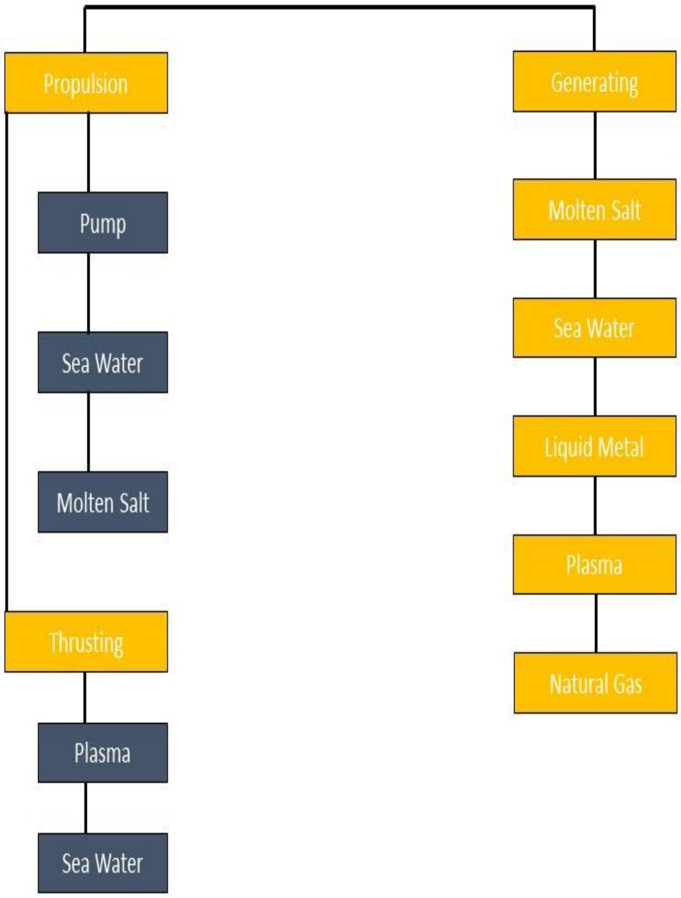

Figure 1. Applications of MHD fluid flow.

Magnetohydrodynamics (MHD) finds applications in astrophysical research, power generation, and space propulsion. Magnetic field regulation of conductive fluids is known as MHD. It explains phenomena like the formation of stars and accretion disk development and makes it possible to generate power from fluid movement effectively enough to drive spacecraft without the need of propellants. In addition to its applications in Maglev trains and geophysical fluid dynamics, MHD is crucial for controlled fusion research.

The determinations of examination are as follows:

- ❖ In magnetohydrodynamic (MHD) flow across a stretching sheet, what is the effect of adding radiative influences on the dynamics of heat and mass transfer?
- ❖ In the shooting method study of heat and mass transfer in MHD flow over a stretching sheet, what are the important parameters taken into account?
- ❖ How do changes in the Schmidt number, Prandtl number, and magnetic field strength impact the transfer rates in MHD flow with radiative effects?
- ❖ What new information does the research offer for creating MHD systems in engineering applications that are more sustainable and effective?
- ❖ How does the shooting approach help us comprehend the intricate interactions that occur in MHD flow over a stretching sheet between fluid behavior, magnetic effects, and thermal-mass exchange?

2. Mathematical Formulation

The study considers radiation in addition to a continuous, 2D magnetohydrodynamic (MHD) circulation across a stretched sheet of a viscous, incompressible fluid. This means examining the behavior of the fluid in a two-dimensional plane under the effect of thermal radiation, fluid viscosity, and magnetic fields. The stretching sheet influences the creation of the border layer and the flow characteristics by acting as a boundary condition as shown in figure 2. The investigation also looks at how radiation affects thermal energy transmission in the fluid. Here, the equations of continuity, momentum, energy, and concentration are as follow:

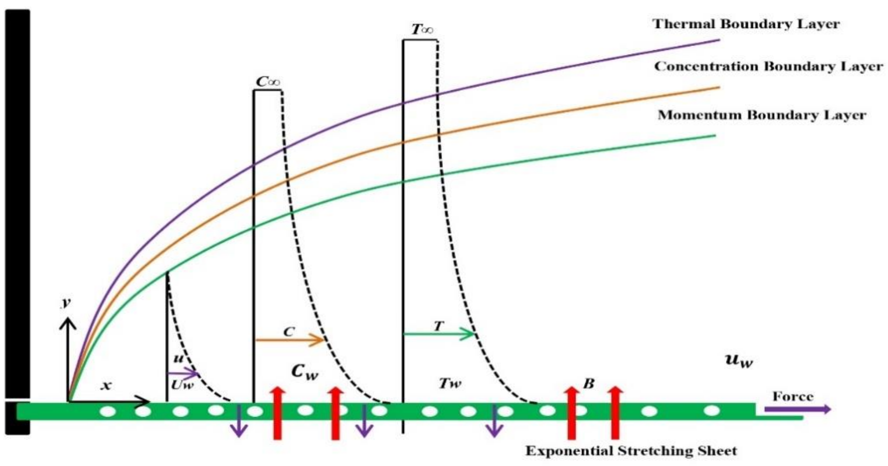

Figure 2. Problem Schematic Diagram.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2}{\rho}u, \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_P}\frac{\partial q_r}{\partial y}, \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}. \quad (4)$$

The radiation Rosseland approximation is defined in (5):

$$q_r=\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y}. \quad (5)$$

In above mathematical expression k^* is the coefficient of absorption and Stefan-Boltzmann constant is σ. We can write T^4 as the temperature as a linear function and by utilizing the Taylor series while excluding higher order components produced:

$$T^4\approx 4T_\infty^3-3T_\infty^4. \quad (6)$$

By using (5) & (6), equation (3) becomes:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma {T_\infty}^3}{3\rho c_P k^*}\frac{\partial^2 T}{\partial y^2}. \quad (7)$$

with suitable boundary conditions

$$\begin{gathered}u=U_w(x), v=0, T=T_w=T_\infty+T_o e^{x/2L}, C=C_w=C_\infty+C_o e^{x/2L}, at\ y=0,\\ u\to 0, T\to T_w, C\to C_w, as\ y=0,\end{gathered} \quad (8)$$

Dimensionless similarity variables are introduced as follows to make the mathematical analysis simpler as shown below

$$\begin{gathered}u=U_o e^{x/L}f'(\eta), v=-\left(\frac{\gamma U_o}{2L}\right)^{1/2}e^{x/2L}\{f(\eta)+\eta f'(\eta)\}, \theta(\eta)=\frac{(T-T_\infty)}{(T_w-T_o)}, \phi(\eta)=\frac{(C-C_\infty)}{(C_w-C_o)}, \eta\\ =y\left(\frac{U_o}{2\gamma L}\right)^{1/2}e^{x/2L}.\end{gathered} \quad (9)$$

Utilizing (9), the following nonlinear ordinary differential equations are found:

$$f'''(\eta)+f(\eta)f''(\eta)-2\big(f'(\eta)\big)^2+Mf'(\eta)=0, \quad (10)$$

$$\left(1+\frac{4}{3}R\right)\theta''(\eta)+P_r\big(f(\eta)\theta'(\eta)-f'(\eta)\theta(\eta)\big)=0, \quad (11)$$

$$\phi''+Sc\big(f(\eta)\phi'(\eta)-f'(\eta)\phi(\eta)\big)=0. \quad (12)$$

The updated boundary conditions are

$$f'(0)=1, f(0)=0, \chi=1, \theta(0)=1, \phi(0)=1, at\ \eta=0,$$
$$f'(\infty)\rightarrow 0, \theta(\infty)\rightarrow 0, \phi(\infty)\rightarrow 0, \chi(\infty)\rightarrow 0, as\ \eta=0. \quad (13)$$

Equations (9) and (10-12) contain the following parameters:

$M=\frac{2\sigma B_0^2 L}{\rho U_o}$: Magnetic parameter,

$Pr=\frac{\gamma}{\alpha}$: Prandtl Number,

$Sc=\frac{\gamma}{D}$: Schmidt Number,

$R=\frac{4\sigma T_\infty^3}{k^* \rho c_p \alpha}$: Thermal radiation parameter.

Skin resistance (Cf_x), the number of Nusselt coefficients (Nu_x), and Sherwood (Sh_x).

$$C_f=\frac{2\tau_w}{\rho U^2{}_w}, \qquad Nu_x=-\frac{xq_w}{T_w-T_\infty}, \qquad Sh_x=-\frac{xm_w}{C_w-C_\infty}. \quad (14)$$

Where the heat flow of the wall (q_w), mass transfer (m_w), and shear stress of the wall (τ_w) are supplied by:

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, q_w=-\left(\frac{\partial T}{\partial y}\right)_{y=0}, m_w=-D_B\left(\frac{\partial C}{\partial y}\right)_{y=0}. \quad (15)$$

Utilizing (9), Local Sherwood number, local Nusselt number, and local Skin friction are

$$f''(0)=\frac{C_f}{\sqrt{\frac{2}{Re_x}}\sqrt{\frac{x}{L}}}, -\theta'(0)=\frac{Nu_x}{\sqrt{\frac{Re_x}{2}}\sqrt{\frac{x}{L}}}, -\phi'(0)=\frac{Sh_x}{\sqrt{\frac{Re_x}{2}}\sqrt{\frac{x}{L}}}. \quad (16)$$

where $Re={}^{U_w x}/_{\gamma}$ is local Reynolds number.

3. Implementation of BVP4C

Bvp4c (Boundary Value Problem with 4th Order Accuracy) is another helpful resource for resolving boundary value problems in magnetohydrodynamics (MHD). The system equations (10-12) can be precisely solved by integrating the particular boundary conditions (11) into bvp4c. Additionally, the shooting approach can be used for validation. Using this approach, the boundary condition estimations are iteratively improved until the solutions converge with the required level of precision. This iterative method provides an additional level of verification for solving MHD boundary value problems, functioning as a supplementary mechanism to the bvp4c solver as shown in figure 3.

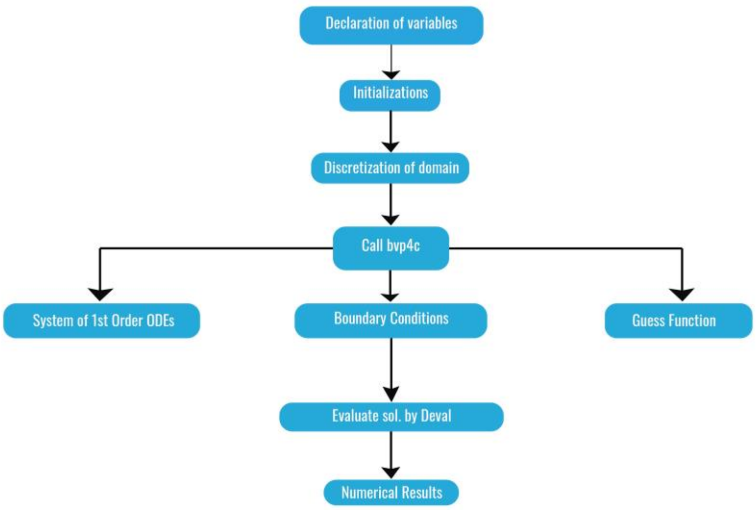

Figure 3. Flow chart of the bvp-4C method.

The governing set of nonlinear ordinary differential equations (ODEs) provided here undergo simultaneous transformation into a system of first-order differential equations.

$$y_1=f, y_2=f', y_3=f'', y_4=\theta, y_5=\theta', y_6=C, y_7=C', \quad (17)$$
$$F_1=y_2, F_2=y_3, F_3=-y(1)y(3)+2(y(2))^2+My(2), \quad (18)$$
$$F_4=y_5, F_5=\frac{1}{\left(1+\frac{4}{3}R\right)}[-Pr(y_1 y_5+y_2 y_4)], \quad (19)$$
$$F_6=y_7, F_7=-Sc(y_1 y_7+y_2 y_6). \quad (20)$$

The boundary condition undergoes a transformation.

$$y_1=0, y_2=1, y_4=1, y_6=1 \text{ at } \eta\rightarrow 0, \quad (21)$$

$y_2 = 0, y_4 = 0, y_6 = 0$ as $\eta \to \infty$.

To derive three initial values, represented as principles of $y_3(0), y_5(0), y_7(0)$ which are not explicitly provided within this context, we utilize the multiple-shot approach. Subsequently, in the forthcoming section, we delve into numerical findings that meticulously examine the effects of different physical elements on the transport of mass and heat towards the movable permeable plumb surface, employing the bvp4c technique.

4. Results and Discussion

We thoroughly examined the impacts of different parameters on fluid dynamics and heat transmission processes in the results section. This required a thorough analysis of the consequences of the Schmidt number, Prandtl number, and magnetic parameter on temperature, concentration profiles, velocity, and temperature. The intricate relationships between heat transfer, mass transfer, and fluid dynamics that occur in magnetohydrodynamic flows over stretching sheets were better understood as a result of these studies.

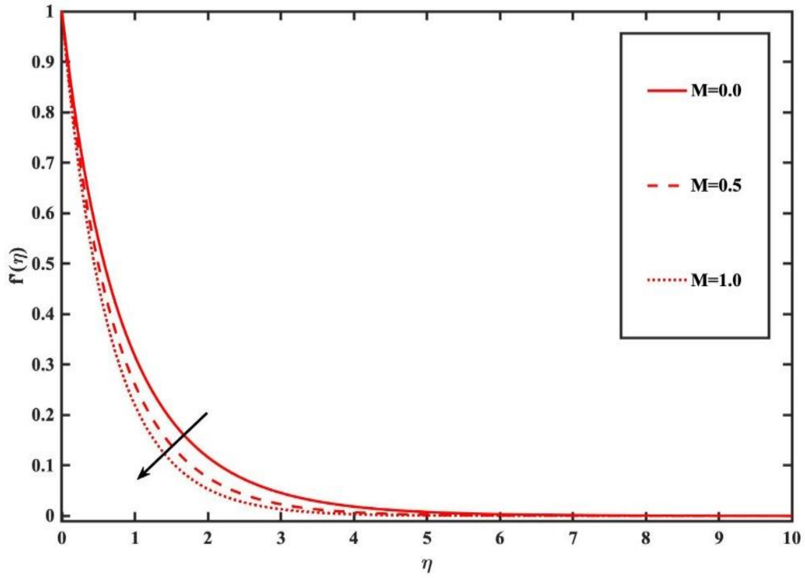

Figure 4. Velocity profile for various magnetic parameter values.

The result of the magnetic parameter (M) on the non-dimensional velocity field is seen in Figure 4. The graph indicates that velocity decreases as the M factor rises. This phenomenon is caused by the force known as Lorentz, which is produced by the magnetic field. The Lorentz force acts as an impediment to the fluid's motion as the value of the magnetic parameter increases, resulting in a reduction in velocity. According to this physical interpretation, larger values of the M parameter are associated with stronger Lorentz force resistance, which in turn affects fluid motion and lowers the total velocity.

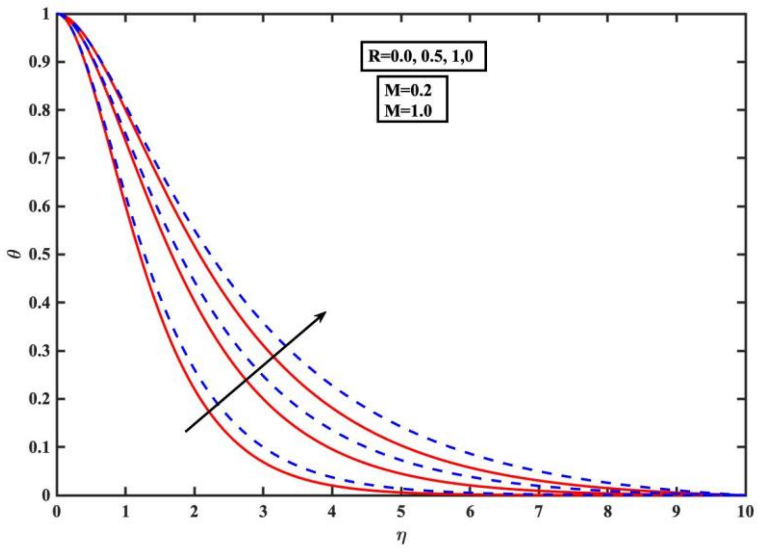

Figure 5. Temperature profile for various magnetic parameter values.

Figure 5 shows the dimensionless temperature field's depicted impacts of various M and R parameter values. The graph shows that as M increases, the temperature distribution rises. The Lorentz force arises with an increase in the magnetic parameter, which raises the temperature and thickens the thermal boundary layer. Furthermore, Figure 4 shows that the dimensionless temperature increases with increasing R value. This happened as a result of improving the R parameter, which aids in improving the temperature field by modifying the energy transmission to the surrounding fluid. Reduces the pace at which heat escapes the sheet as a consequence.

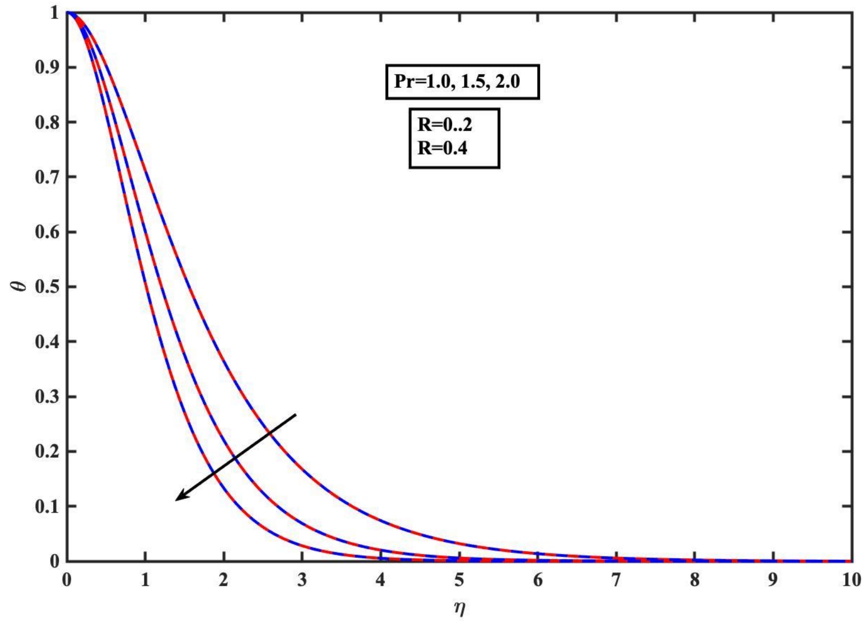

Figure 6. Temperature profile for various Prandtl number values.

A graphic depiction of the effects of various Prandtl number and radiation parameter values on the dimensionless temperature field can be found in Figure 6. The graph shows that thermal diffusivity decreases with increasing Prandtl number levels. As a result, quicker heat diffusion away from the heated surface is made possible by the thermal barrier layer becoming thinner. The heat capacity increases as a consequence, increasing the amount of heat transfer. Understanding and adjusting these characteristics is crucial for engineering and scientific applications, as demonstrated by the association between the Prandtl number and thermal diffusivity that has been established and its consequences for heat transfer processes.

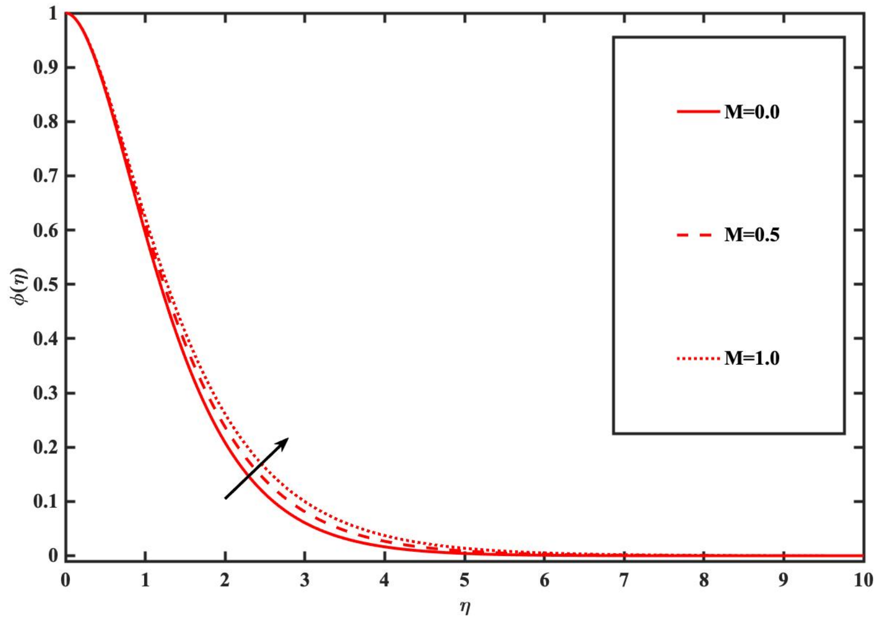

Figure 7. Concentration profile for various magnetic parameter values.

The result of the magnetic parameter on the dimensionless concentration profile is seen in Figure 7. The graph shows a pattern whereby higher M parameter values cause the concentration distribution throughout the border layer to increase. As a result, because of the border layer of concentration thicker structure, the rate of mass transfer is lowered. Surprisingly, it is shown that when M values rise, the characteristics of the concentration distribution show qualitative equivalency with the temperature distribution. This result emphasizes how mass and heat transfer mechanisms in magnetohydrodynamic flows are interrelated, underscoring the significance of taking both into account in thorough assessments of fluid dynamics.

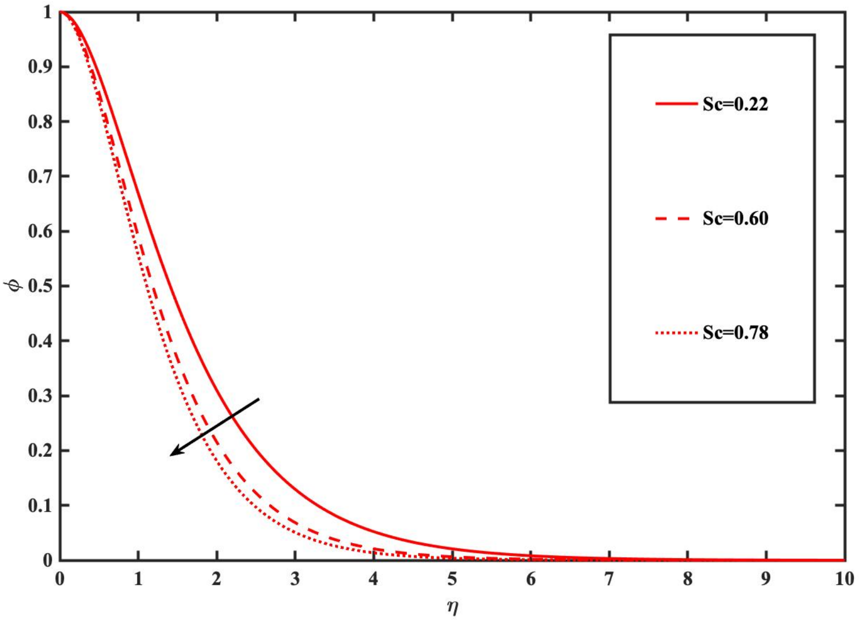

Figure 8. Concentration profile for various Schmidt number values.

The concentration distribution is shown to be reduced when the Schmidt number upsurges in Figure 8. This pattern is explained by the fact that greater Sc numbers cause a decrease in particle diffusivity, which in turn causes the concentration level to drop. Consequently, when the thickness of the concentration boundaries layers diminishes, the mass transfer rate rises. Remarkably, Prandtl number influences the thickness of the thermal boundaries in a manner similar to Schmidt number influences the concentration border layer thickness. This analogy offers important insights into fluid dynamics and boundary layer behavior while highlighting the interdependence of mass and heat transfer events.

4.1 Validation of the present study

A comparison of local Nusselt numbers with published statistics, $-\theta'(0)$ is made in order to verify the accuracy and efficacy of the shooting technique used. The Runge-Kutta technique results, which are shown in Table 1, show a remarkable agreement with the results obtained in this work utilising the shooting method. This relationship increases trust in the study's conclusions by highlighting the shooting method's dependability and accuracy in forecasting local Nusselt values.

Table 1. Analyzing Local Nusselt Number Differences, $-\theta'(0)$ for different values of R, M, Pr with Sc=0.

R	M	Pr	Seini and Makinde [42]	Khalili et al., [43]	Present study
0	0	1	0.9548	0.9550	0.9552
		2	1.4715	1.4714	1.4715
		3	1.8691	1.8690	1.8644
0	1	1	0.8615		0.8611

Table 1. Comparative analysis of the research gap by taking physical factors into account.

	Wang et al., [36]	Gbadeyan et al., [37]	Alharbi et al., [38]	Al Nuwairan et al., [39]	Abo-Dahab et al., [40]	Ali et al., [41]	Present Study
Year	(2022)	(2020)	(2021)	(2022)	(2021)	(2024)	
Viscous fluid	No	No	No	No	No	Yes	Yes
MHD	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Shooting Method	No	No	No	No	No	No	Yes

Conclusions

This work explores the phenomena of heat and mass transport in the context of radiation-affected magnetohydrodynamics (MHD) boundary layer movement across an exponentially extending sheet submerged in a viscous, incompressible fluid. By skillfully employing similarity variables, the challenging partial differential equations are simplified to feasible nonlinear ordinary differential equations. We then address these equations numerically using the shooting approach. Results show that fluid velocity drops and fluids temperature and concentration notably rise with increasing magnetic parameter. These findings offer some intriguing new information. Moreover, higher radiation parameters are found to be correlated with higher temperatures; yet, temperatures show a negative tendency as Prandtl numbers rise. Furthermore, it is seen that the liquid concentration dispersion declines as Schmidt number values increase. Furthermore, higher radiation parameters lead to increased temperatures, although temperatures decrease with rising Prandtl numbers. Additionally, the distribution of fluid concentration diminishes as the Schmidt number values increase.

Nomenclature

Surface temperature T_w
Surface concentration C_w
Ambient temperature T_∞
Ambient concentration C_∞
Magnetic field $B(x) = B_0 e^{\frac{x}{2L}}$
Thermal diffusivity α
Kinetic viscosity γ
Fluid temperature in boundary layer T
Radiative heat flux q_r
Specific heat c_p
Fluid density ρ
Molecular diffusivity D
Stefan- Boltzmann constant σ
Absorption coefficient k^*
Stretching velocity $U_w(x) = U_0 e^{\frac{x}{L}}$
Reference velocity U_0
Characteristics length L
Magnetic parameter $M = (2\sigma B_0^2 L)/\rho U_0$
Prandtl number $Pr = \frac{\gamma}{\alpha}$
Schmidt number $Sc = \frac{\gamma}{D}$
Thermal radiation parameter $R = \frac{4\sigma T_\infty^3}{k^* \rho c_P \alpha}$
Reynold number $Re = {U_w x}/{\gamma}$
Shear stress $\tau_w = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}$
Rate of heat transfer $q_w = -\left(\frac{\partial T}{\partial y}\right)_{y=0}$
Mass flux $m_w = -D_B \left(\frac{\partial C}{\partial y}\right)_{y=0}$

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Conflicts of Interest: The authors declare no conflict of interest.

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