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Forecasting Mortality Rate of Pakistan by Using Lee-Carter Model

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Abstract

This research has been focused-on to forecasting the mortality rate of Pakistan population by using Lee-Carter Model (LCM). In order to accomplish this purpose, the mortality data of Pakistan for male and female from 2000-2016 are utilized. LCM parameters were estimated from the data of different age groups with year wise. This LCM model can also be shown in logarithm form as ($\log m_{x,t}$). Moreover, Singular Value Decomposition (SVD) technique is also used to compute the mortality index and least squares estimate of LCM. It can lead to predict the life expectancy and mortality rates of the data used. All the resultant outcome of this research including life expectancy and forecast mortality rate has produced an auto regressive integrated moving average (ARIMA) model. In addition, a comprehensive comparison is made among male and female mortality rate and their life expectancy periods. The computation and estimation of different parameters have been done by using R software. The mortality rates of male and female population have been predicted for next ten years. The current research will be helpful for devising different policies and planning related to population of Pakistan.

Keywords: *Lee-Carter (LC) Model, Mortality Modeling, Forecasting, Life Expectancy.*

Introduction

The Lee-Carter Model (LCM) is crucial for predicting mortality, fertility, migration, marriage, and divorce rates. Employing year-wise projections, LCM forecasts annual fertility and mortality rates for specific age groups, aiding economists and policymakers in strategic planning. These predictions hold significant social and economic implications, guiding budgetary decisions and estimating life expectancy based on fertility rates. Utilizing statistical methods, such as the LCM, is imperative for anticipating a country's future population and fostering its development (Guralnik *et al.*, 1988).

In analyzing mortality rate fluctuations, the Lee-Carter Model (LCM) employs parameters to predict stochastic mortality. These parameters are estimated through matrix conversion of annual age-grouped mortality rates. The correlation is high, determined by LCM parameter estimation using the first principal component analysis (Lee and Miller, 2000; Brouhns *et al.*, 2002).

The Lee-Carter model, initially designed for life expectancy, is now frequently used to predict mortality rates at various ages. Smoothing methods reduce effective parameters, enhancing predictive accuracy and addressing model risk in pension and annuity pricing (Ganser *et al.*, 2003) . Utilize SVD to reduce dimensionality for stochastic mortality projections and employ LCM for forecasting. It's integral for social protection, healthcare planning, and institutional pricing in retirement income systems (Renshaw and Haberman, 2006).

This research aims to forecast Pakistan's mortality rate using the Lee-Carter Model (LCM). Data

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from 2000-2016 for males and females will be employed, estimating LCM parameters for various age groups year-wise. The model, expressed as log[m(x,t)], will utilize the Singular Value Decomposition technique and the least square method to predict the mortality index accurately.

This research utilizes stochastic modeling to forecast mortality rates by simulating 100 iterations, calculating confidence intervals for age-specific death rates. The outcomes, including life expectancy, mortality rates, and life tables, will be integrated into an autoregressive integrated moving average (ARIMA) model. A detailed gender-based comparison of mortality rates and life expectancy will be conducted using R software, providing comprehensive estimates for various parameters and predicting Pakistan's mortality rate.

Objectives

- Utilize the Lee-Carter Model for forecasting the mortality rate.
- Estimate the mortality index and derive mortality rates for the population of Pakistan.

Review of Literature

In recent years, forecasting has gained significance in vital statistics. Numerous methods with diverse criteria have been proposed for effective modeling of future data. The Lee Carter Model, introduced (Freedman and Peters, 1984) in econometric modeling, has been pivotal in mortality forecasting. Building upon this, simulations for real-life scenarios and extensions to vital rates, including fertility fluctuations control, have been explored. The forecasting method, utilizing cohorts, establishes mortality intervals as outlined by(Alho, 1990).

In demographic research, forecasting the random components of the cohort model is crucial. This model predicts the random components of a nonlinear function of vital rates across age groups and both sexes. Challenges arise in forecasting both-sex mortality and fertility variations. Addressing this, Alho (1990) employs a linear function joint distribution approach for stochastic calculation simulation. Another issue in literature pertains to vital rates, tackled by Wilmoth (1990) through the Lee Carter Cohort, decomposing rates into specific age, period, and cohort components.

Alho (1990; 1991) identified forecasting components and assumptions through random parametric modeling. Parameters were estimated from previous data, obtaining intervals for forecasting. Data analysis from 1972 to 1985 for males and females aged 45-70 demonstrated 95% accuracy of suggested mortality rate intervals. Alho (1990) extended work through Lee-Carter Model (LCM) for demographic components, determining mortality rates via Poisson regression. Lee and Carter (1992) calculated mortality rates for both genders under LCM's Poisson regression, constructing life expectancy tables with Poisson log-bilinear model estimation. Criticisms arose, including narrow probability bands and the model's suboptimal treatment of biomedical information (Bell, 1997; Lee and Miller, 2000). (Lee and Miller, 2001) compared parameter estimation under the Lee Carter Model, addressing social security factors. (Booth *et al.*, 2002) highlighted LCM's flexibility in real-life scenarios, emphasizing linear changes over time. (Brouhns *et al.*, 2002) presented a criterion for LCM, crucial for demographical forecasting. (Booth *et al.*, 2002) structured ARIMA forecasting, creating premiums. (Renshaw and Haberman, 2003) compared methods for gender and age-specific deaths.

Koissi et al. (2006) applied LCM to forecast Nordic countries' mortality rates, estimating parameters using Decomposition, Least Square, and Maximum Likelihood Estimation. LCM generated mortality rate and life expectancy forecasts, incorporating uncertainty through residuals and bootstrap techniques.

Bisetti and Favero (2014) explored the impact of longevity risk on pension systems, measuring risks by the difference between estimated and total costs of increasing old-age pensions. The potential redistributive effects of longevity risk were discussed, advocating for a market to address uncertainties. Zili et al. (2018) employed the Lee-Carter model, utilizing SVD and ARIMA methods to forecast mortality rates in Indonesia. Their R software-based approach demonstrated low error (0.3%) in predicting mortality rates for several years. Saikia et al. (2020) assessed the feasibility of LCM in forecasting Assam's mortality rate, utilizing Lee and Carter's stochastic mortality model fitted through SVD. The ARIMA model projected mortality index variations until 2030.

Materials and Methods

Demographic data on mortality rates in Pakistan are systematically gathered from reputable sources such as the World Health Organization and the United Nations. The data includes mortality rates and estimated population figures, meticulously organized according to age structures. The comprehensive information derived from these sources ensures reliability and accuracy in the assessment of Pakistan's demographic landscape, contributing to informed analyses and decision-making processes.

Lee-Carter Model

(Lederman, 1969) proposed a two-parameter model utilizing principal component analysis (PCA) to construct life tables. The method estimates mortality by deriving a single parameter for age-specific distribution within the Lee-Carter Model, yet it does not forecast fertility rates, as indicated by(Bozik and Bell, 1989).

The notation m (x, t) represents the central death rate for a specified age interval, with x indicating the age group and t denoting the time period or year. The model focuses on fitting the logarithm of the central death rate to capture the underlying patterns and trends.

$$log\, m_{x,t} = \alpha_x + \beta_x k_t + \varepsilon_{x,t} \quad (1)$$

Central death rates denote log($m_{x,t}$) is stated x, a_x function overall mortality for age period is a bilinear expression b_x, $k_t$with Error term $\varepsilon_x \sim N(0, \sigma^2)$ which return back the give age specific variable by the model not captured. Bilinear expression composed by mortality index $k_t$of the measure of mortality over specific time interval. All calculations and results were derived using the Demography and Forecast package in R.

Results and Discussion

Mortality rates for the male population of Pakistan from the years 2000 to 2016 are detailed in World Health Origination website. The data illustrates an overall decline in male mortality rates across specific age groups during this period.

In Figure 1, Pakistan male mortality rates reveal a low trend between ages 5 to 45, spiking after 50. Minimal log death rate change occurs between ages 10 to 40, indicating low mortality. Figure 1 for Pakistan female mortality rates displays low rates from 10 to 55 and a rising trend after 55. Death rates barely change between ages 10 to 40 but increase significantly beyond 55. These findings are based on fitted log death rates for Pakistan's male and female populations from 2000 to 2016.

Figure 1: Male and Female Death Rate of Pakistan Population 2000-2016.

Fitting of the LC Model and Parameters estimation to Pakistan Mortality Rate

Table1: $\hat{a}_x$ And $\hat{b}_x$ For Pakistan Mortality Rate Based on Year Wise Data 2000-2016.

	$\hat{a}_x$		$\hat{b}_x$	
Age Group	**Male**	**Female**	**Male**	**Female**
0-4	-3.13475	-3.170937	0.11509	0.0790825
5-9	-6.907755	-6.866982	0.0000000	0.037623156
10-14	-6.907755	-6.907755	0.0000000	0.0000000
15-19	-6.866982	-6.866982	0.11643	0.0376231
20-24	-6.255381	-6.500022	0.09979	0.2109402
25-29	-6.214608	-6.581568	0.00000	0.220304825
30-34	-5.999950	-6.255381	0.21594	0.035435341
35-39	-5.792221	-6.071503	0.04832	0.107490493
40-44	-5.495209	-5.724531	0.05200	0.064995162
45-49	-5.117653	-5.377074	0.05214	0.059481129
50-54	-4.732047	-4.932852	0.05903	0.050308218
55-59	-4.308232	-4.501576	0.03662	0.040518778
60-64	-3.880989	-4.028950	0.04476	0.033590890
65-69	-3.433473	-3.551496	0.03804	0.023879578
70-74	-2.982353	-3.064149	0.03894	0.012874010
75-79	-2.523239	-2.569382	0.03243	0.004326622
80-84	-2.060298	-2.078096	0.03048	-0.00584886
85+	-1.458933	-1.445154	0.01999	-0.01262528

In Table 1, the Lee-Carter estimated parameters for Pakistan's mortality rates from 2000 to 2016 are presented. The values for α (age-specific parameters) and β (year-specific parameters) provide insights into mortality trends. For instance, in the male population, the age group 0-4 shows a significantly negative impact on mortality, while the age group 20-24 exhibits a positive influence. Similarly, for females, the age group 85+ has a positive impact, suggesting an increase in mortality with age. These parameters aid in understanding the complex relationship between age, time, and mortality in the given population.

Table 2: Male and Female Mortality Index ($\hat{k}$) For Pakistan Mortality Rate Based on Year Wise Data 2000-2016.

Year	**2000**	**2001**	**2002**	**2003**	**2004**
Male	0.9155959	0.7026932	0.70268588	0.66974990	0.47225665
Year	2005	2006	2007	2008	2009
Male	1.81202579	0.38049656	0.40289851	0.26429501	0.05604697
Female	2.53485097	0.68002338	0.44646055	-0.0795802	-0.20862621
Year	**2010**	**2011**	**2012**	**2013**	**2014**
Male	0.14533077	-0.6106285	-1.0221760	-0.8762314	-0.91361233
Year	**2015**	**2016**			
Male	-0.94929211	-1.5397731			
Year	**2000**	**2001**	**2002**	**2003**	**2004**
Female	1.70744519	1.59271805	1.66929914	1.66929919	1.47386669
Year	**2010**	**2011**	**2012**	**2013**	**2014**
Female	-0.67600420	-1.2336679	-1.2665393	-1.5413613	-1.65460875
Year	**2015**	**2016**			
Female	-2.01490119	-2.5571882			

Estimate the year-wise mortality index ($\hat{k}$) for males and females from 2000 to 2016. Males show a declining trend, while females exhibit a downward trend. A comparison indicates a slower decline in female mortality compared to males. Table.2 illustrates the mortality index differences in the declining years for male and female populations.

Forecast Mortality Rate From 2017-2026

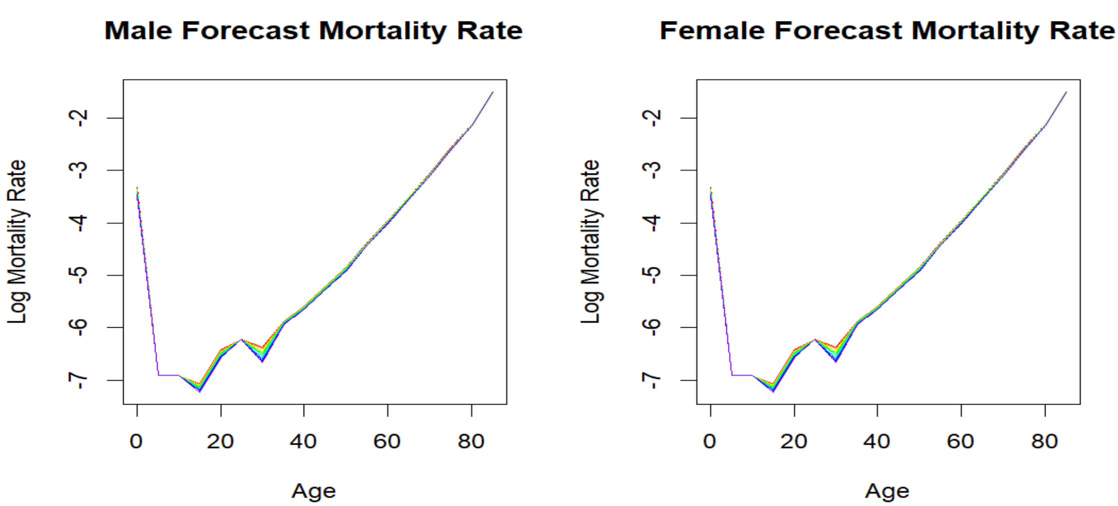

Figure 2: Forecast Log Mortality Rates Male and Female Population 2017-2026.

Figure 2 illustrates the forecasted mortality rates for the male and female populations from 2017 to 2066. In the 0-10 age group, log death rates are notably high due to increased child mortality. For ages 15-30, mortality rates exhibit varied fluctuations. Beyond age 40, a substantial upward trend in mortality rates is observed for both genders.

Forecast Life Expectancy Male and Female from 2017 to 2026

Table 3: Forecast Life Expectancy Male and Female from 2017 to 2026.

Years	Male	Female
2017	64.37	66.22
2018	64.49	66.35
2019	64.61	66.48
2020	64.73	66.61
2021	64.85	66.74
2022	64.96	66.86
2023	65.08	66.99
2024	65.19	67.11
2025	65.31	67.23
2026	65.42	67.34

Table 3 presents the forecasted life expectancy for both males and females in Pakistan from 2017 to 2026. The average male life expectancy is projected to be 64.90, while for females, it is estimated to be 66.80. The lower life expectancy in Pakistan is primarily attributed to a high incidence of hepatitis B and C infections, particularly affecting individuals between 20 to 30 years. However, increased awareness, governmental initiatives, and accessible treatments have contributed to a gradual rise in life expectancy over the years, reflecting positive strides in combating these health challenges.

Fitted Mortality Index Kt with ARIMA Model for Male and Female.

Table 4: Fitted Mortality Index Kt with ARIMA Model for Male and Female.

	ARIMA (0,1,1) with drift for Male	ARIMA (0,1,0) with drift for Female
AIC	25.29	30.4
BIC	27.61	31.95

The table .4 presents the fitted mortality index (kt) using the ARIMA model for both male and female populations. The ARIMA (0,1,1) model with drift was employed for males, resulting in an AIC (Akaike Information Criterion) of 25.29 and BIC (Bayesian Information Criterion) of 27.61. For females, the ARIMA (0,1,0) model with drift was used, yielding an AIC of 30.4 and BIC of 31.95. Lower AIC and BIC values indicate better model fit. These ARIMA models with drift are tailored to capture the temporal patterns in mortality index, providing valuable insights into the mortality trends for both genders.

Forecasted Values Mortality Index $\widehat{k}$With 95% CI 2017-2026

Table 5: Forecasted values mortality index $\hat{k}$ with 95% CI 2017-2026.

	Male			Female		
Years	**Forecast**	**Hi 95**	**Lo 95**	**Forecast**	**Hi 95**	**Lo 95**
2017	-1.3273	-2.3526	-0.3020	-2.4981	-3.7292	-1.2670
2018	-1.3273	-2.5512	-0.1034	-2.4981	-4.1529	-0.8434
2019	-1.3273	-2.7217	0.0672	-2.4981	-4.4883	-0.5079
2020	-1.3273	-2.8736	0.2190	-2.4981	-4.7749	-0.2214
2021	-1.3273	-3.0118	0.3573	-2.4981	-5.0292	0.0329
2022	-1.3273	-3.1395	0.4850	-2.4981	-5.2602	0.2639
2023	-1.3273	-3.2588	0.6043	-2.4981	-5.4733	0.4770
2024	-1.3273	-3.3712	0.7166	-2.4981	-5.6721	0.6759
2025	-1.3273	-3.4777	0.8231	-2.4981	-5.8592	0.8630
2026	-1.3273	-3.5791	0.9245	-2.4981	-6.0364	1.0402

Table 5 displays the forecasted values of mortality index with 95% confidence intervals for both male and female populations in Pakistan from 2017 to 2026. The negative values indicate a projected decline in mortality rates over the next decade. The forecasting process involved utilizing the best-fitted model to estimate mortality index values for each year. The consistent decline in mortality rates for both genders reflects the decreasing nature of the mortality index (kt) over the specified period, suggesting positive trends and improved life expectancy for the population in Pakistan.

Forecasted Life Expectancy Age wise 2017-2066.

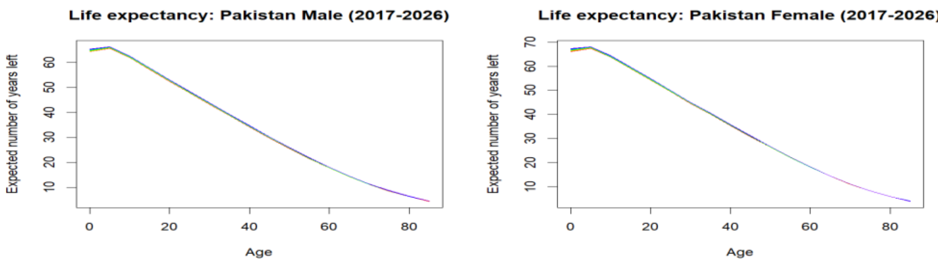

Figure 3: Forecast Life expectancy age wise of male and female 2017-2026.

Figure 3 illustrates the forecasted age-wise life expectancy for the male and female populations from 2017 to 2026. For males, there's a notable decline in life expectancy, particularly in the 15-40 age group, attributed to factors like hepatitis B, C, and tobacco use. Females, too, exhibit a slight decline after age 5, with higher life expectancy in the 0-10 age range. The figures highlight critical trends in life expectancy, emphasizing the impact of health challenges in Pakistan.

Conclusion

This research focused on forecasting the mortality rates of the Pakistan population using the Lee-Carter model (LCM). Mortality data from 2000-2016 for males and females were employed to estimate LCM parameters, revealing male variation at 50.3% and female variation at 68.4%. The estimated mortality patterns exhibited an increasing trend between age groups 25 years to 85+, emphasizing a noteworthy decline in mortality index during 2000-2016, more prominent in males. Forecasted mortality rates for 2017-2026 indicated an overall improvement, particularly in male mortality rates.

Stochastic modeling forecasted mortality indices using a random walk with drift, showing a decline spread during 2017-2026, with males experiencing a higher decline compared to females. ARIMA (0,1,1) models with drift were used for male mortality indices, while ARIMA (0,1,0) models were applied for females. Life expectancy calculations for 2017-2026 highlighted a positive trend, with significant improvements in females compared to males, supported by the forecasted 95% confidence interval.

The study used R software for comprehensive estimations, producing outcomes such as life expectancy, mortality rates, and life tables. A notable conclusion is the need for improvement in Pakistan's healthcare system, as factors like the shortage of medicines, lack of medical facilities, and the spread of diseases contribute to a lower life expectancy (64 to 66 years) compared to other regions. Addressing these issues is crucial to enhancing life expectancy in Pakistan and fostering overall public health. The research not only contributes to mortality forecasting methodologies but also sheds light on critical health challenges in the country.

Note: Please Incorporate the Study Gap

The study gap is that Pakistan has not yet utilized the Lee-Carter model for mortality rate analysis.

Additionally, the Lee-Carter model has not been employed for fertility rate and migration rate assessments in Pakistan.

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