Kurdish Studies
Mar 2024
Volume: 12, No: 2, pp.5609-5628
ISSN: 2051-4883 (Print) | ISSN 2051-4891 (Online)
www.KurdishStudies.net

Received: 02 January 2024 Accepted: 24 March 2024
DOI: https://doi.org/10.58262/ks.v12i2.416

A Comparative Analysis of Arithmetic Word Problem Solving in Singaporean and the Kingdom of Saudi Arabia Mathematics Textbooks

Dr. Mansour Saleh Alabdulaziz[1], Hanadi Mashal Aljoaid[2]

Abstract

This study is aimed at comparing the attributes and frequency of arithmetic word problems (AWPs) included in textbooks used in mathematics classes at the primary level in Singapore and Saudi. The main focuses of the research are on: the amount of arithmetic word problems; the variation in problems with regard to their multiplicative or additive structures as well as their semantic-mathematic substructures; and the amount and characteristics of illustrative materials that accompanied the arithmetic word problems. While AWP tasks were more prevalent in the textbooks used in Singapore, the findings indicated that AWPs had the same comparative diversity in the mathematics textbooks in both countries. Additionally, the textbooks used for mathematics in both Singapore and Saudi Arabia placed emphasis on the structures categorised as (additive) combine 1 and (multiplication) simple rate with regard to AWPs. In particular, there was a greater proportion of illustrations that depicted the problems' mathematical structures within the textbooks in Singapore, which assisted learners with the AWP problem-solving process. Based on the outcomes, it is recommended that the distinctions among students from countries with lower and higher levels of achievement are not mainly due to the frequency and variation in tasks that involve solving AWP problems included in textbooks, but are primarily attributed to accompanying illustrations.

Keywords: *word; arithmetic word problem solving, a comparative analysis, illustrations, mathematics textbooks, primary education.*

Introduction

As stated by Vicente et al. (2020), the textbook is a core component of mathematics teaching, and comprises a fundamental aspect of the curriculum that largely dictates the content of in-class teaching (see also Hiebert et al. 2003). Therefore, they are influential in terms of mathematics teaching standards and the ability of students to learn (Sievert et al. 2019; Heinz et al. 2009). Empirical research has demonstrated that particular characteristics of the way in which textbooks are designed affects the proficiency of students in mathematics, which includes word problem solving (Chang & Silalahi 2017; and Sievert et al. 2019).

The concept of "mathematics problem solving" has been interpreted in a variety of different ways. Schoenfeld (1992) distinguished between standard activities targeted at providing the opportunity for students to practice a specific mathematical method, which is certainly a highly

[1] Department of Curriculum and Instruction, College of Education, Imam Abdulrahman Bin Faisal University, P.O. Box 2375, Dammam 31451, Saudi Arabia, Emails: malabdulaziz@iau.edu.sa or, dr.malabdulaziz@hotmail.com
[2] Graduate student, College of Education, Imam Abdulrahman Bin Faisal University, Saudi Arabia

beneficial skill, and the ability to solve complex of challenging problems (Schoenfeld, 1992). Arithmetic word problems (AWPs) are specific problems in mathematics that are regarded as being an important method of instructing learners on how to employ mathematics, find coherence in normal situations as well as general metacognitive and heuristic abilities that are required for solving complex problems (in terms of Polya 1945; Verschaffel et al. 2020). Multiple countries have started to focus on such problems as students find it difficult to find solutions, and numerous educators have stated that the level of performance in this topic is the lowest (Novotná and Chvál 2018).

Nevertheless, as revealed by TIMSS (Trends in International Mathematics and Science Study), the level of proficiency in mathematics of Singaporean pupils is superior to that of those from other countries (Mullis et al. 2020). One of the reasons for this could be the textbooks used in mathematics lessons, which offer adequate instructional content for students to learn how to solve problems (Vicente et al. 2020). However, no previous studies have analysed and compared the attributes and frequency of arithmetic word problems (AWPs) included in the textbooks used for teaching mathematics at the primary school level in Saudi Arabia with different countries. The current study aimed at investigating whether textbooks used for teaching mathematics in countries with high levels of performance like Singapore have increased effectiveness in promoting the learning and reasoning of students compared with students in countries where the performance levels are average like Saudi Arabia.

Problem of Study

It is widely acknowledge that "the primary goal of mathematics instruction should be to have students become competent problem solvers" (Schoenfeld 1992, 334). In fact, the ability to solve problems is largely regarded as the foundation of educational curricula and the basis of theoretical frameworks used for the purpose of assessing student achievement levels internationally. Nonetheless, the extent to which problem-solving skills are developed in students differs significantly according to the specific country. As stated in a recent report published by the Trends in Mathematics and Science Study (TIMSS) (Mullis et al., 2020), 55% of students in the fourth grade in Singapore were sufficiently proficient in mathematics that they were capable of solving the most difficulty problems set for them. Contrastingly, solely 1% of students in Saudi Arabia had achieved a similar level. Such differences could be explained by the manner in which parents in Singapore view the education of their children (e.g., the extent to which families are involved in school education, the societal significant of education, and the importance assigned to meritocracy) (Rao et al. 2010). Nevertheless, different factors are involved in teaching mathematics, such as the approach taken to integrate mathematics problems into teaching plans, which can facilitate our understanding of why there are differences in performance levels (Chapman 2006).

The focus of the present research on textbooks used for teaching mathematics in primary schools, which are frequently adopted by teachers to foster students' abilities to solve mathematics problems (Depaepe et al. 2009; Hiebert et al. 2003). Additionally, research findings indicate that textbook content could influence the levels of proficiency students are able to achieve (e.g., Fagginger Auer et al. 2016; Heinze et al. 2009; Törnroos 2005; Siegler & Oppenzato 2021; Sievert et al. 2019, 2021). In particular, the study examined arithmetic word problems (AWPs) due to the fact that they are regarded as being important methods of fostering the advancement of primary school students' problem-solving abilities (Verschaffel et al. 2020).

Research Objective

The aim of this study was to compare how mathematics textbooks from Singapore and Saudi promote the skills to solve arithmetic word problems.

Theoretical Framework

The approach taken by students for solving arithmetic word problems has been described using various models. For instance, for example, Verschaffel et al. (2000) proposed that when solving arithmetic word problems, genuine or superficial approaches can be taken. The former permits learners to solve all kinds of word problems, regardless of the extent to which they are complex and difficult, and incorporates a comprehension of the structure of the mathematical problem through the use of mathematical reasoning. The latter approach consists of moving directly from the data to the operation, then the result. Solving word problems in this way could be beneficial for basic problems where the solution can be found relatively easily without the need for significant reasoning, but it is less suitable for problems with increased complexity in which various kinds of reasoning are required for a solution to be found (the operationalisation of basic and complicated problems is shown below). Some of the strategies in which a superficial approach is adopted include: (i) the actions implied in the problem text are directly modelled using physical tools, like fingers or blocks (Riley & Greeno 1988) and (ii) the "key word strategy", where students approach AWPs from a symbolic perspective using numerals, in which certain words (e.g., "won") are taken as cues for the selection of arithmetic operations (e.g., addition), with no consideration of different (con)textual inputs (Hegarty et al. 1995; Verschaffel et al. 1992).

The model of Verschaffel et al. (1992) certainly emphasises the notion that that different kinds of AWPs exist, which can be more or less challenging to represent and resolve, thus requiring various different problem-solving approaches. Similarly, the theoretical framework underpinning TIMMs connects achievement in various kinds of word problems to overall mathematics achievement levels. Pupils whose mathematics achievement levels are categorised as low to intermediate are only capable of solving basic word problems. Nevertheless, a higher or more advanced level of mathematics achievement means that students are capable of solving problems that necessitate an in-depth conceptual understanding and/or metacognitive or heuristic thinking.

Various criteria have been proposed for determining AWPs' degree of complexity (Daroczy et al. 2015). From such criteria, it has been identified that the problem's semantic-mathematical structure largely determines the problem's degree of complexity (Carpenter & Moser 1984; Greer 1992; Heller & Greeno 1978; Vergnaud 1991) in addition to the approaches needed to find a solution (Carpenter et al. 1981). It is possible to categorise one-step additive AWPs according to the widely recognised classification suggested by Heller and Greeno (1978) and Carpenter and Moser (1984) as problems consisting of as change, compare, combine, and equalize. Moreover, distinct subcategories can be determined according to the unknown set and the relation (additive or subtractive) among the sets included in the problem (see Fig. 1).

According to the findings of a similar study, multiplicative AWPs inherently have increased complexity (Verschaffel et al. 2007), the classification of which can be made based on the semantic-mathematical structure (albeit with a less recognised classification). For example, according to the given operation as well as the unknown set, there are four distinct kinds of multiplicative AWPs, which can be divided into distinct subcategories: (1) rate (or equal groups), (2) multiplicative comparison (or scalars), (3) Cartesian product, and (4) rectangular matrix (Greer 1992; Vergnaud 1991, see Fig. 2).

Distinct levels of processing are also reflected in this classification. According to Verschaffel et al. (2000), the process of solving certain AWPs can be relatively simple, whereas other problems necessitate a greater level of understanding. It is possible to solve basic AWPs (e.g., Change 1, according to Fig. 1) utilising the term "won" and direct modelling of the activity depicted in the word problem (i.e., combining the $5 in my possession to the $3 I earned). On the other hand, it is also possible to use "won" as a clue to suggest that the two numbers given in the problem should be added. Nevertheless, solutions to complex AWPs can only be found by reasoning on the relationships between numbers and the application of particular conceptual knowledge, like the comprehension of part-whole relationships or proportional reasoning. For instance, as shown in Figure 2, it is not possible to apply direct modelling or use a basic keyword approach to solve multiplicative multiple-rate AWPs; it is necessary for the learner to unravel the complexity of such AWPs structures before the problem can be solved through the application of solution strategies such as the rule-of-three strategy.

Based on the aforementioned theoretical framework, the assumption is made that the ability to sufficiently develop a credible method by which AWPs can be solved is dependent on tackling various different AWPs that incorporate problems with varying levels of complexity (Despina & Harikleia 2014; Schoen et al. 2021). This notion concurs with the variation theory of learning (Marton 2015). This theory postulates that is necessary for learners to be exposed to different kinds of problems in order to identify and concentrate on the core elements of solving problems. Put differently, for generalising the concept of what solving AWPs comprises and developing sufficient strategies for solving such problems, learners must discern the commonalities linking the solutions to distinct kinds of AWPs. By becoming proficient in this area, students will no longer concentrate on superficial cues (e.g., the keyword strategy) as a standard means of finding solutions to all kinds of arithmetic word problems. It is important to note that even problems in which only one-step subtraction or addition are required incorporate varied semantic-mathematical frameworks, meaning that errors can frequently occur when applying the superficial problem-solving approach.

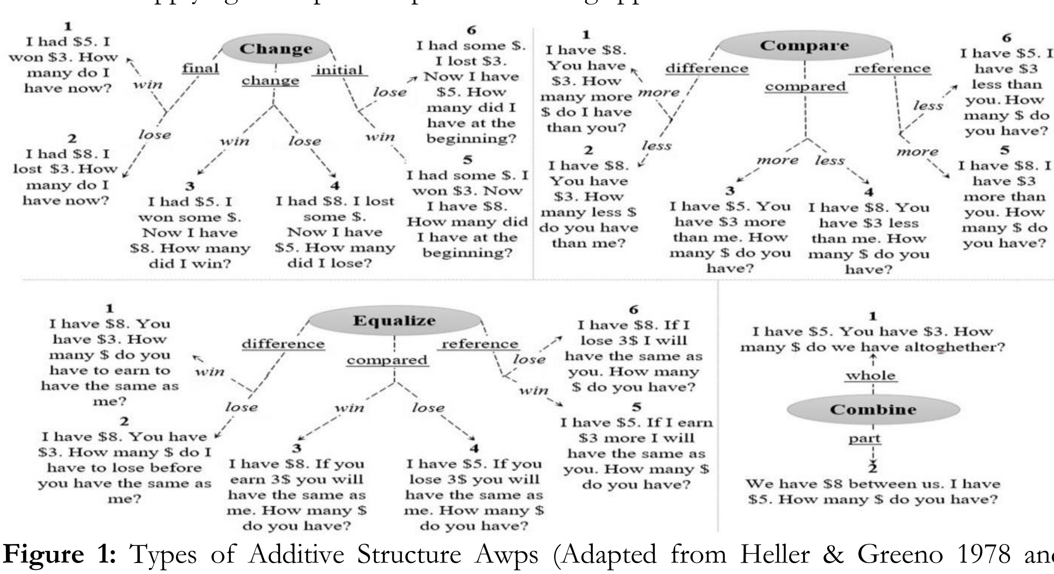

Figure 1: Types of Additive Structure Awps (Adapted from Heller & Greeno 1978 and

Carpenter & Moser 1984).

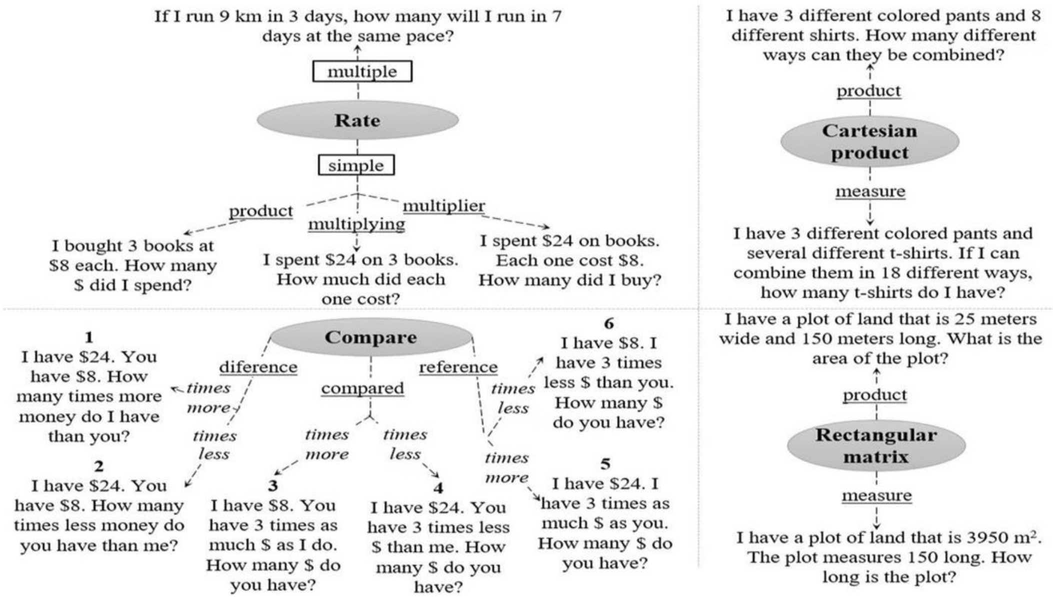

Figure 2: Types of Multiplicative Structured Awps (Adapted from Greer 1992 and Vergnaud 1991).

The Impact of Illustrations on AWPs 2.1

Scientists focused on learning and teaching the topic of mathematics have largely acknowledged the role that representations play in the process of solving problems. Recently, the benefit of representations, largely described as an arrangement of symbols, images or tangible things that represent a different entity, (Dewindt-King & Goldin 2003), has been emphasised by different research domains in mathematics teaching.

Empirical studies have explored the roles that different types of illustrations play from various theoretical viewpoints. For instance, Elia and Philippou (2004) investigated the purpose of images in solving word problems. According to the categorisation of Carney and Levin (2002), Elia and Philippou (2004) identified four functions: decorative pictures, representational pictures, organisational pictures, and informational pictures. Decorative pictures are images that are not related to the problem and/or how it can be solved; rather, they essentially give the page an aesthetic appearance. On the other hand, representational pictures depict the problem either partially or in full such as by showing an image that represents (a pertinent) aspect of the scene the problem describes Organisation pictures are images that offer guidelines that allow (a portion of) the solution process to be identified, such as through the use of schematic depictions of the problem's mathematical framework. Compared to the other three categories mentioned above, informational pictures provide details that are critical for comprehending and/or solving the problem, such as critical numerical data that the problem statement does not include. Elia and Philippou (2004) identified that for problems to be solved effectively though the use of pictures, the picture's function is important. They found that the process of solving problems can be significantly impacted by informational, organisation and representational pictures, while the effect of decorative pictures is minimal. Different researchers have also proposed that the contribution of decorative pictures to the process of solving mathematical problems is minimal (Agathangelou et al. 2008). However, there is no consensus regarding the effect of representational pictures. For example, while Elia and Philippou (2004) determined that such pictures positively affect the process of solving word problems, Agathangelou et al. (2008) identified that students' competence in solving mathematical word problems was not impacted by representational pictures.

The findings of various researches indicate that showing learners schematic organisational representations of problems is highly beneficial for their ability to solve them (Xin, 2019). Conversely, the evidence regarding the effect of representational pictures is not conclusive (Hegarty & Kozhevnikov 1999; Vicente et al. 2008). The notion that the performance of students can be enhanced via schematic depictions of AWPs' semantic mathematical structures has received significant empirical reinforcement from what is known as schema-based instruction (SBI), an approach used to teach how to solve problems that places emphasis on the problem's semantic and mathematical structures (Marshall 2012). In this method, the benefits of relational diagrams are integrated with schema theory. According to the stage theory of development proposed by Bruner (1973), the assumption is made that the enactive or iconic presenting of problems in mathematics can resolve issues connected with problems that students cannot understand using symbols. The model method (Kho 1987), which is regarded as being one of the core components of teaching mathematics in the Singaporean education system, is a type of SBI (Kaur 2019). Structured processes are used as part of this method such that learners are instructed on the visualisation of abstract mathematical relationships as well as their different problem structures via schematic depictions (Ferrucci et al. 2008) prior to the word problem being solved (Kaur 2019). Therefore, in the model method, the illustrations employed can be regarded as being organisational illustrations according to Ellia and Philippou's (2004) classification as they denote the problem's mathematical structure and promote the problem-solving process.

Mathematics Textbooks - Opportunity to Learn

While the way in which textbooks are utilised differs significantly, it is undeniable that they have a significant influence on mathematics teaching (Rezat & Strässer 2014). Rezat and Strässer (2014) contended that textbooks are elements of a didactical tetrahedron, where didactical situations are shaped in combination with the teacher, learners, and mathematics. As stated by activity theory (Rezat 2006), textbooks are perceived as being cultural artifacts used by teachers and pupils within a culturally-mediated environment (i.e., classrooms) to accomplish a specific goal (e.g., to teach students how problems are solved), thus allowing the "subject-mediating artifact-object" triad to be established.

In fact, the performance of students improves in subjects that textbooks cover to a large extent (Schmidt et al. 2001; Törnroos 2005). For example, students' learning of fundamental arithmetic principles is enhanced when they are exposed to more relevant activities (e.g., Sievert et al. 2021); likewise, when textbooks place increased emphasis on strategies for solving problems, it is more likely that students will use them (e.g., Fagginger Auer et al. 2016; Heinze et al. 2009, Sievert et al. 2019, 2021). Indeed, subjects that are excluded from textbooks are generally not included in the classroom teaching and learning process (Schmidt et al. 1997). For example, the proficiency of students in solving specific mathematical problems involving fractions and decimals is frequently reduced as they are infrequently included in textbooks (Siegler & Oppenzato 2021).

In this regard, it is likely that textbooks in countries with higher performance like Singapore could provide students increased exposure to solving more varied AWPs compared with students in countries where the performance is in the low to mid range.Moreover, insufficient opportunities to practice such problems and or specific kinds of schematic depictions could be obstacles in the learning process (Siegler & Oppenzato 2021). Research indicates that the textbooks used in countries where performance is higher contain a more varied and balanced distribution of AWPs incorporating additive and multiplicative elements in a variety of types of problems compared with textbooks used in other places like the US (e.g., Schoenfeld 1991; Stigler et al. 1986) and Spain (e.g., Orrantia et al. 2005: Tárraga et al. 2021; Vicente et al. 2018, 2022). Evidence also shows that the illustrations in textbooks used in countries with higher performance have greater richness (i.e., manipulatives, pictures, tables, models, graphs and diagrams) that could facilitate the process by which students understand mathematical structures (Chang & Silalahi 2017). For instance, Mayer et al. (1995) determined that compared with the US, textbooks used in Japan included illustrations with greater relevance along with instructional techniques that were more meaningful, emphasising the various methods by which problems can be represented in the form of pictures, words, and symbols.

Method

Procedure

In the present research, all AWP examples contained within the mathematics textbook series targeted at primary school students published by the primary publishers (with respect to the distribution proportion throughout the education system) in Saudi Arabia and Singapore – the Ministry of Education 2022 education and the Marshall Cavendish (MC) 2022 education, respectively – were reviewed. The AWPs included in the analysis related to activities that: (i) incorporate a verbal account of actual or contrived scenarios by designing a mathematical problem that necessitated a minimum of the four fundamental arithmetic operations, and (ii) could be categorised as having one of the research-based multiplicative or additive AWP frameworks illustrated in Figure 1 and Figure 2. AWPs that were determined to be examples in which all the steps required to solve the problem were also included. For tasks where the criteria described above were not satisfied, including where the student was required to solve arithmetic operations or use calculations for the purpose of solving scenarios in which the problem was not sufficiently placed in context (e.g., "calculate the amount of flowers by multiplying", according to a picture depicting five vases where each one contained four flowers), were not categorised as being AWPs. Different kinds of mathematical problems (e.g., algebraic, statistical and geometrical problems like those involving the calculation of perimeters) were disregarded when conducting analysis for the present study,

Categories of analysis: AWPs vs. other Mathematical Activities (OMAs)

For the purposes of the research, an "activity" was regarded as being a specific task or group of associated tasks that comprised an individual teaching exercise on a page of the textbook, denoted by the title, number or guidelines displayed above the activity or through different design elements. Such activities required students to either solve one or multiple problems or they were guided in solving them, where the problems generally necessitated calculations or different mathematical skills All activities included in the textbooks were allocated to one of two categories: (1) Activities requiring AWPs to be solved and (2) other mathematical activities (OMAs). Due to the fact that the former incorporated one or multiple AWPs, there were more AWPs than activities in which AWPs needed to solved. For the most part, OMAs consisted of exercises, along with some mathematical problems not considered to be AWPs. Based on the analysis of the respective textbooks, a total of 14,185 activities were found (7,985 in MC and 6,200 in Ministry of Education), where 3,278 were categorised as being AWP activities (2,127 in MC and 1,151 in Ministry of Education). In the subsequent analysis, only these activities identified as being AWP activities were included. The overall amount of AWPs included in these activities was determined to be 4,992 (2,642 in MC and 2,350 in Ministry of Education).

Categories of Analysis: Semantic/Mathematical Structure

The problems were assigned to one of two categories according to whether their structures were additive or multiplicative in nature. The initial stage involved the decomposition of multistep AWPs into the individual parts from which they were formed with regard to the semantic/mathematical structure. In the present research, the analysis involved a total of 7,680 semantic/mathematical structures (3,828 in MC and 3,852 in Ministry of Education).

Additive AWP types: AWPs had structures that were reflective of problems specifically involving addition or subtraction. As suggested by Heller and Greeno (1978) and Carpenter and Moser (1984), each problem was assigned to a specific category from the options of change, compare, combine, and equalise. Twenty distinct subcategories were identified according to the unknown set as well as the extent relations (subtractive or additive) among the sets included in every AWP (see Fig. 1 above).

Multiplicative AWP types: Such problems consistently included division or multiplication. As suggested by Greer (1992) and Vergnaud (1991), multiplicative AWPs were divided into four categories: (1) rate (or equal groups), (2) multiplicative comparison (or scalars), (3) Cartesian product, and (4) rectangular matrix. Fourteen subcategories were identified according to the unknown set as well as the arithmetic operation (division or multiplication) needed for the given AWP to be solved (see Fig. 2 above).

Categories of Analysis: Illustrations

The analysis only included illustrations that were explicitly displayed in the textbooks. In the research, it was examined whether the use of illustrations facilitated students' understanding of the problem's mathematical structures as well as whether they offered information in terms of the problem's wording. Therefore, in the context of the present research, the classification proposed by Ella and Philippou was adapted and three different kinds of illustrations were differentiated:

a) Figurative: Such illustrations are pictorial in nature and portray a constituent, portion or the entire problem situation; however, (i) they do not reference the solution in any way (similar to the decorative illustrations described by (Elia & Philippou 2004), (ii) there is no provision of numerical information, and (iii) the mathematical structure of the problem is not displayed (which is similar to representational illustrations described by Elia & Philippou 2004).
b) Informational: These could take the form of pictorial illustrations, tables or graphs that include information required for the problem to be solved. In other words, they are information sources that substitute the problem text.
c) Organisational: Such illustrations are schematic in nature and either partially or wholly depict the problem's mathematical structure, thus allowing students to comprehend how the

problem sets are mathematically related. Additionally, numerical information pertaining to the problem is included in such illustrations. "Bar modelling" used in textbooks in Singapore is an example of this type of illustration.

Data Coding

Initially, the proportion of activities in the respective textbooks that specifically required students to solve AWPs (as stipulated in the process) was determined. Then, to identify the extent to which the textbooks had incorporated diverse semantic-mathematical structures, every one-step problem was examined to determine whether it was multiplicative or additive in nature, and it was then allocated to one of the aforementioned subcategories (see Figs. 1 and 2). AWPs in which multiple steps were required to find the solution were broken down into separate constituent parts, which were categorised individually such that there were more structures than the overall amount of AWPs that were distinguished. Lastly, for analysing the purpose of illustrations, the proportion of AWPs that had accompanying illustrations was determined. On this matter, it is important to note that a minimal percentage of problems included in the textbooks published in Singapore (1.89% of the overall amount) and Saudi Arabia (4.44%) had two accompanying illustrations (both organisational and figurative illustrations in the Singapore cases, while the instances in the Saudi textbook included figurative and informational illustrations). Subsequently, the classifications of the illustrations were determined from aforementioned options of figurative, informational, or organisational. When an AWP had twin illustrations, it was scored twice.

The process of categorising the problems (AWP activities vs. OMAs, the semantic-mathematical structure of the AWP, and the illustration category) was preliminarily conducted by the researcher in conjunction with 11 arbitrators who were experts in mathematics teaching until such time that the criteria required to ensure the reliability of the analysis had been determined. After this had been achieved, the AWPs' semantic-mathematical structures were analysed along with different categorisations. *Data analysis*

A quantitative approach was adopted to conduct the analysis based on the volume and kind of data obtained. Due to the characteristics of the data, non-parametric statistics were utilised in the research. Pearson's chi-square test (or Fisher's exact test) was employed to ascertain the existence of a relationship between the publisher of the textbook and (1) the frequency of exercises dedicated to solving AWPs, (2) the diversity of semantic/mathematical structures, and (3) the kinds of illustrations. For the purpose of comparing particular kinds of AWPs among publishers, z-tests with Bonferronni adjustment for multiple comparisons were conducted. For effect size verification, the Cramér's V was used, which shows whether it is small (0.1), moderate (0.3) or large (0.5), as suggested by Cohen (1988).

To further demonstrate the variation in the semantic-mathematical structures employed by the respective publishers, the method proposed by Petersson et al. (2021) was adopted, and Lorenz curves were estimated to identify if there was a balanced distribution of the AWP types. It important to emphasise that more variation is not an implication of the balanced distribution of AWPs as the frequency of certain types could be higher. Lorenz curves are frequently employed for describing and comparing how wealth is distributed and income inequality. It describes the relationships between the cumulative proportions of population (%Pi) and income (%Yi), such that if the proportion of income that each population segment has is identical ($P_i = Y_i; \forall i$), a line at 45° line will be generated (known as the "perfect equity line"). Equity denotes homogeneity in the way in which the categories are distributed. Therefore, because the Lorenz curve portrays a relative cumulative distribution and depicts the proportional entirely of data after sorting or ordering, it is possible to use it to demonstrate the cumulative distribution of every type of AWP as a fraction of the overall number of AWPs within the textbook. In the present research, the Lorenz curve nearing the diagonal indicates that the distribution of problem structures is more equal, suggesting that there is an equal representation of such textbooks in the textbooks.

Hypotheses

Based on both the abovementioned theoretical framework as well as the outcomes of past research with respect to the correlation between textbooks for mathematics and the degree of mathematics proficiency (in addition to the results of international assessments), the following hypotheses are proposed:

Firstly, taking into account that students in Singapore have increased competence in solving AWPs compared to those in Saudi Arabia (Mullis et al. 2020) as well as that past research has demonstrated that competence in particular mathematics concepts is correlated with aspects that are more commonly practiced (Tornroos 2005; Schmidt et al. 2001), the expectation was that textbooks on mathematics in Singapore would include an increased percentage of exercises requiring AWPs to be solved of the overall amount of mathematics exercises (Hypothesis 1).

Secondly, considering that exposure to a variety of different problems can improve student learning (Siegler & Oppenzato 2021), the expectation was that textbooks used in Singapore would incorporate a more extensive or diverse selection of AWPs based on their semantic-mathematical structures (Hypothesis 2a). Moreover, it was assumed that there would be a more balanced distribution of AWPs according to their structural type (multiplicative or additive) in the textbooks from Singapore. In the case of the textbook from Singapore, the Lorenz curve would be nearer to the line of perfect equity (Hypothesis 2b).

Thirdly, considering that the teaching of mathematics in the Singaporean education system is based on the model method (Kho 1987) as well as that the said model demonstrates the process of visualising abstract mathematical relations to students via schematic depictions (Ferrucci et al. 2008), it was assumed that textbooks in Singapore would include a greater percentage of organisational illustrations compared to those used in Saudi Arabia (Hypothesis 3).

Results

Frequency of AWP-Solving Activities

A total of 7,985 mathematics activities were included in the textbook published in Singapore, 2,127 of which required AWPs to be solved. On the other hand, there were 6,200 such activities in the textbook published in Saudi Arabia with 1,151 involving AWPs. The results of the chi-square test showed that in line with the predictions of Hypothesis 1, the percentage of activities involving AWPs in the Singaporean textbooks exceeded that of the textbooks in Saudi Arabia (AWPs vs. OMAs), $\chi2$ (1, n = 14,189) = 90.50, $p < .001$. It can be observed that there was a small effect size (0.08).

Problem Variability

The analysis included 7,680 structures in total. From the 3,828 and 3,852 fundamental structures covered by the analysis of the textbook published in Singapore and Saudi Arabia, additive structures accounted for 53.3% and 55.3%, respectively.

Additive Structures

As shown in Table 1, a total of 18 distinct categories of structures were identified in the mathematical problems contained in the textbooks from Singapore (90% of the available additive structures), whereas there were 17 distinct kinds of structures in the mathematical problems identified in the textbooks from Saudi Arabia (85%). The results of the Fisher's exact test corroborated these findings regarding the variety of additive structures ($p > .66$).

Table 1: Frequencies in Absolute Numbers and Percentages of Every Kind of Additive Structure according to Publisher.

Type of structure		Publisher			
		Marshall Cavendish		Ministry of Education	
		N	%	N	%
Change	1	153	6.9	126	6.4
	2	284	12.9	350	18.9
	3	87	3.9	28	1.4
	4	43	1.9	19	0.9
	5	33	1.5	4	0.2
	6	33	1.5	8	0.4
Compare	1	119	5.4	149	7.6
	2	83	3.7	100	5.1
	3	99	4.5	64	3.2
	4	86	3.9	39	2.0
	5	34	1.5	2	0.1
	6	31	1.4	2	0.1
Combine	1	725	32.0	799	41.0
	2	355	16.2	253	12.9
Equalize	1	20	0.9	3	0.1
	2	3	0.1	1	0.0
	3	1	0	0	0
	4	0	0	0	0
	5	0	0	0	0
	6	2	0.0	0	0
Total		2,191	100	1,947	100

However, the results of the chi-square goodness-of-fit test revealed that the distinct types of structures were not distributed in an even manner in either the Singaporean or Saudi textbook MC: $\chi2$ (17) = 4,413, $p < .001$; Ministry of Education: $\chi2$ (14) = 5,300, $p < .001$). An in-depth examination of Table 1 reveals that despite the fact that the textbook in Singapore includes 90% of the 20 distinct structures, students did not have the same level of exposure to the various different mathematical structures. Indeed, the frequency with which the majority of structure types were included in the textbooks in Singapore was either significantly under or over 113 (5% of the overall amount of structures detected); this represents the anticipated frequency for all types of structures in a model where all structures are equally probable, thus giving students the same level of exposure to all the different possible structures (i.e., similar observed frequencies). The results for the textbook from Saudi Arabia were analogous.

It is important to note that in combination, the three kinds of fundamental structures - Combine 1 and 2 and Change 2, which are regarded as being problems with low medium levels of difficulty (Combine 1 and Change 2) (Combine 2) in the opinions of previous researchers (Nesher 1981; Rathmell 1986; Riley & Greeno 1988; Riley et al. 1983) - accounted for 61.1% of the additive structures in the textbooks from Singapore and 72% of those from Saudi Arabia, whereas most of the other categories were highly infrequent in both countries' textbooks. This finding calls into question the perceptions of problem variation in textbooks for mathematics. It can be observed that the textbook from Saudi Arabia contained minimal examples of problems categorised as Change 5 and 6 (0.2 and 0.4), suggesting that problem distribution in the textbooks was less equal compared to the textbook from Singapore.

It was found that the textbooks differed to a certain extent with respect to particular kinds of structures. For example, problems categorised as being Change 3, 4, 5 and 6 and Compare 4, 5 and 6 had an increased frequency in the textbook from Singapore compared to the one from Saudi Arabia. It is important to note that in line with the previously mentioned studies, these problems are considered to have medium and high levels of difficulty. On the other hand, an increased amount of problems categorised as being Combine 1, Change 2 and Compare 1 and 2 problems were found in the textbook from Saudi Arabia, which are regarded as having a low level of difficulty.

Multiplicative Structures

As can be observed in Table 2, mathematics problems included in the textbooks from Singapore and Saudi Arabia included 11 of the 14 kinds of multiplicative structures. The variation in multiplicative structures was corroborated by the results of the Fisher's exact test (p = 1).

Table 2: Frequencies in Absolute Numbers and Percentages for all Kinds of Multiplicative Structure according to Publishers.

Type of structure		**Publisher**			
		Marshall Cavendish		Ministry of Education	
		N	%	N	%
Rate	Multiplication-rate	592	38.1	973	58.9
	Division-rate partition	300	19.4	248	15.0
	Division-rate quotition	235	15.3	244	14.7
	Multiplication rate multiple	261	16.9	53	3.2
Compare	Multiplication compare "times more"	51	3.5	39	2.3
	Multiplication compare "times less"	1	0.1	2	0.2
	Division compare reference unknown "times more"	25	1.7	1	0.0
	Division compare compared set unknown "times less"	6	0.4	15	0.9
	Division compare scalar unknown "times more"	3	0.2	1	0.0
	Division compare scalar unknown "times less"	0	0	0	0
Cartesian product	Cartesian Product-product	0	0.0	1	0.0

	Cartesian Product-measure	0	0.0	0	0
Rectangular matrix	Rectangular matrix-product	57	3.8	63	3.8
	Rectangular matrix-measure	8	0.6	10	0.6
Total		1539	100	1650	100

Similar to the findings above regarding additive structures, the data in Table 2 does not show significant variation in the distribution of multiplicative structures across the different textbooks. The Lorenz curve estimations revealed a marginally more balanced distribution of distinct kinds of multiplicative structures in the textbook from Singapore compared to the one from Saudi Arabia, albeit with a significantly unbalanced distribution in both cases (chi-square goodness-of-fit test: MC: $\chi 2$ (11) = 2,527.13, $p < .001$; Ministry of Education: $\chi 2$ (11) = 4,800, $p < .001$).

It is important to note that three kinds of structures categorised as being simple-rate problems accounted for 72.8% and 88.6% of the multiplicative structures in the textbooks from Singapore and Saudi Arabia, respectively.

While the textbooks differed with regard to particular kinds of structures to a certain extent, the effect sizes were minimal. For example, the multiple-rate problems had a greater frequency than simple-rate problems in the case of Singapore, and likewise included more division compare reference unknown "times more" and more rate partition problems in division structures than the Saudi textbook.

Overall, neither Hypothesis 2a or Hypothesis 2b are supported by these results with regard to both additive and multiplicative structures.

Illustrations

In the textbook from Singapore, there was a smaller percentage of AWPs with accompanying illustrations 45% (n = 1,193) compared to the textbook from Saudi Arabia 50.% (n = 1, 301). With regard to these illustrations' purpose (as shown in Table 3), the results of the chi-square difference test showed that the publisher and kind of illustration were correlated. The textbook from Singapore contained a significantly higher percentage of organisational illustrations targeted at promoting and bringing clarity to the problem's mathematical structure; on the other hand, an increased proportion of figurative illustrations were contained in the textbook from Saudi Arabia (see Table 3). Moreover, an examination of the proportion of illustrations whose function was not figurative, such as illustrations that were more informational in nature as they presented information supplementary to the problem's wording or depicted the problem's mathematical structure, revealed that they were more prevalent in the textbook from Singapore (82.3%) compared to the one from Saudi Arabia (51.4%). Hypothesis 3 is supported by these findings.

Table 3: Frequency in Absolute Numbers and Percentages of Kind of Illustration according to Publisher.

Type of illustration	Publisher			
	Marshall Cavendish		Ministry of Education	
	N	%	N	%
Figurative	220	17.4	664	48.5
Informational	602	47.8	659	48.2
Organizational	435	34.5	44	3.2
Total	1257		1367	100

The overall Amount of Illustrations Exceeds the Amount of Problems Accompanied by

Illustrations due to the Awps with Twin Accompanying Illustrations.

Discussion

The variation in the TIMSS results suggests that the countries differ significantly in terms of their learning experiences. This, in the present research, the investigation focussed how Singapore and Saudi Arabia, two countries that differ with respect to their international assessment scores, use mathematics textbooks to enhance students abilities to solve arithmetic word problems. . To achieve the study objectives, the research examined the frequency of AWP activities, the diversity of semantic-mathematical structures incorporated into the AWP activities, as well as the kinds of illustrations that complemented the AWPs in textbooks produced by the primary publishers in the two countries. Vicente et al. (2020) stated that due to the fact that textbooks are core components of mathematics education, they largely underpin the curriculum that largely determines the content of in-class teaching (see also Hiebert et al. 2003). Consequently, they are influential in terms of mathematics teaching standards and therefore can impact the ability of students to learn (Sievert et al. 2019; Heinz et al. 2009).

The textbooks designed for teaching mathematics in Singapore are based on the concept that the ability to solve mathematical problems is a core component of learning the subject. This process requires students to acquire and apply mathematics skills and concepts to a broad variety of scenarios, which includes open-ended, non-routine and real-world problems. Hence, the strong proficiency of students in Singapore in terms of solving word problems (Mullis et al. 2020) could be associated (to a certain degree) with the possibility to practice various different types of mathematics problems in the textbook used in Singapore (in line with variation theory, Marton 2015). This implies that variation theory is highly influential on mathematics teaching in the country. Specifically, variations in the AWPs to which children are exposed could give them the chance to not only solve basic problems that can be solved easily (i.e., through the use of a keyword strategy; Hegarty et al. 1995; or direct modelling, Riley & Greeno 1988) but also attempt to solve problems with a higher level of difficulty that necessitate in-depth mathematical reasoning. Moreover, the provision of illustration that promote the comprehension of the problem's semantic-mathematical structure is a fundamental aspect of the approach adopted by the education system in Singapore in the context of the mathematics teaching and learning process (Kaur 2019).

The outcomes revealed that the textbook in Singapore had a greater focus on AWPs compared to the textbook in Saudi Arabia, as there was a higher prevalence of activities requiring AWPs to be solved. Furthermore, in the textbooks used in Singapore, there was increased use of illustrations to accompany the AWPs, which represented the underpinning semantic-mathematical structure. These outcomes applied to both multiplicative and additive problems. Nevertheless, such differences had small effect sizes (other than those pertaining to illustrations), and it is important to note that the textbook series in both countries included an increased amount of OMAs compared to AWPs. Additionally, this research found that the textbooks in Singapore and Saudi Arabia offered analogous variation in types of problems with respect to their semantic-mathematical structures, and both also had insufficient balance in terms of the way in which such structures were distributed. This findings appears to be in agreement with previous studies including those by (Orrantia et al. 2005: Stigler et al. 1986; Tárraga et al. 2021; Vicente et al. 2018). However, the current research is differentiated from those previously mentioned due to various factors. First, while they concentrated on textbooks used at middle and secondary school levels, the present study purely concentrated on those used in primary schools. It is widely acknowledged that the foundations for learning and skills developed in the future are laid during the primary school period due to the fact that the values and skills acquired during this time are the fundamental basis for everything that follows. Therefore, primary education provides a foundation that facilitates future learning, and therefore, selecting an appropriate primary school is critical. Second, the studies were conducted in distinct places, and the different samples included in the analysis could have impacted the outcomes.

However, it was determined that the textbooks used in Singapore and Saudi Arabia were differentiated to a certain extent with respect to particular kinds of AWPs. For example, the textbook from Singapore contained some structures that were not detected (such as Equalize 2 and 6, or Division compare partition "times more") or had very few examples (such as Compare 5 and 6) in the textbook from Saudi Arabia. Additionally, the textbook from Singapore incorporated an increased percentage of more difficult problems (Change 5 and 6 and Compare 5 and 6 with additive structures in addition multiplication rate problems with multiplicative structures which are regarded as being AWPs with increased difficulty) (Greer 1992; Heller & Greeno 1978). The increased percentage of certain kinds of problems in the textbook from Singapore could be attributed to the differences in the two countries' curricular objectives. For instance, multiple-rate problems (comprising multiplicative structures) could introduce students to the process of solving various kinds of ratio problems (Musa & Malone 2012). It is important to note that a specific aim of the curriculum in Singapore is to develop students' abilities to solve ratio problems, but this is not the case in Saudi Arabia.

In the textbooks from Saudi Arabia, illustrations frequently accompany arithmetic problems. The findings of the current research indicated that tasks requiring AWPs to be solved in the Saudi textbooks more frequently had accompanying illustrations, although the difference had a small effect size. Additionally, significant variations in the purpose of illustrations was observed. For example, it was determined that the textbooks in Singapore had a greater proportion of organisational illustrations that facilitated students' learning of the process of solving AWPs via reasoning. When such graphical aids are included, students have the opportunity to make sense of the various mathematical structures underpinning problems with similar wording. This ensures that students do not become dependent on superficial approaches like the use of "keywords" for solving both basic and complex problems. Additionally, these types of illustrations play a role in helping students learn solution strategies that can be applied to all types of problems, irrespective of the degree of semantic-mathematical complexity. In the context of the Singaporean textbook, when basic problems are accompanied by organisational illustrations, while students acquire problem-solving abilities, they are primarily expected to comprehend the functioning and pertinence of such aids such that they are personally capable of applying such organisational tools to problems with increased complexity, both in terms of arithmetic problems at lower grades (in the form of multiplication compare "times more" problems) and different kinds of problems as they become more advanced (i.e., algebra and ratios).

It is important to note that different organisational illustrations indicating how learning how to solve AWPs with varying degrees of complexity is scaffolded by schematic depictions (compare problems in this study), ranging from (easy, additive) Compare 2 problems to the more challenging multiplication problem compare "times more" to algebra problems expressed comparatively. This concurs with the constructivist approach adopted by the mathematics curriculum in Singapore, which is grounded on a concrete-pictorial-abstract (C-P-A) strategy.

It is important to note that the textbooks in Saudi Arabia have a greater percentage of figurative illustrations in comparison to those in Singapore.

Limitations

Despite the meticulous approach taken when preparing this study, various limitations were still confronted, although the outcomes were not negatively impacted. For example, the research purely concentrated on AWPs, as they are regarded as being the primary instrument for fostering the advancement of primary school children's proficiency in solving problems (Verschaffel et al. 2020), and it is the task most frequently used in primary school mathematics textbooks in Singapore. Nevertheless, if more time had been available, ratios and algebra could have been included in the research.

Additionally, only primary school mathematics textbooks were included in the study sample, as they are broadly utilised by educators for promoting the ability to solve mathematics problems (Depaepe et al. 2009; Hiebert et al. 2003). Moreover, findings have indicated that textbook content could be influential on students' degree of proficiency (e.g., Fagginger Auer et al. 2016; Heinze et al. 2009; Törnroos 2005; Siegler & Oppenzato 2021; Sievert et al. 2019, 2021).

Recommendations

Based on the outcomes of this researchers, the following recommendations can be made:

The results indicate that the differences among students from countries with lower and higher achievement levels are not predominantly related to the volume and variation in tasks requiring AWPs to be solved that are included in textbooks, but primarily due to the properties of accompanying illustrations. The fact that the textbooks in Singapore incorporate a greater percentage of problems in which the mathematical structure was represented schematically, which appears to concur with the findings of Vicente et al. (2022). Hence, it is recommended that more problems in which schematic representations of the mathematical structure are presented should be included in the Saudi textbooks.

Suggestions

The researcher suggests that additional research should be conducted in several areas:

1- This study could be replicated and extended to include middle and high school mathematics textbooks with a larger sample size.
2- Further studies need to be conducted to investigate how Singaporean and Saudi children learn to solve problems in mathematics classes.
3- This study focused only on AWPs, but further studies could focus on other types of mathematical word problems, such as those involving algebra and ratios.

References

Agathangelou, S., Gagatsis, A., & Papakosta, V. 2008, September. *The role of verbal description, representational and decorative picture in mathematical problem solving*. Paper presented at the

Conference of Five Cities, Nicosia, Cyprus. Retrieved from http://www.pre.aegean.gr/labs/mathlab/conference2011/icon/ First %20 Conf erenc e% 2 0 of%205cities. Pdf

Bruner, J. S. 1973. *Beyond the information given: Studies in the psychology of knowing*. W.W. Norton.

Carney, R. N., & Levin, J. R. 2002. "Pictorial illustrations still improve students' learning from text". *Educational Psychology Review,* 14: 5–26. https:// doi. org/10.1023/A:1013176309260

Carpenter, T.P., & Moser, J. M. 1984. "The acquisition of addition and subtraction concepts in grades one through three". *Journal for Research in Mathematics Education,* 15 (3): 179–202. https:// doi. org/ 10. 2307/ 748348

Chang, C. C., & Silalahi, S. M. 2017. "A review and content analysis of mathematics textbooks in educational research". *Problems of Education in the 21st Century,* 75(3): 235–251. https:// doi. org/ 10.33225/ pec/ 17. 75. 235

Chapman, O. 2006. "Classroom practices for context of mathematics word problems". *Educational Studies in Mathematics,* 62(2): 211–230. https:// doi. org/ 10. 1007/ s10649- 006-7834-1

Cohen, J. 1988. *Statistical power analysis for the behavioral sciences*. Lawrence Erlbaum Associates.

Daroczy, G., Meurers, K. M., & Nuerk, H. C. 2015. "Word problems: a review of linguistic and numerical factors contributing to their difficulty". *Frontiers in Psychology, 6:* 1–13. https:// doi. org/ 10. 3389/ fpsyg. 2015. 00348

Depaepe, F., De Corte, E., & Verschaffel, L. 2009. Analysis of the realistic nature of word problems in upper elementary mathematics education in Flanders. In L. Verschaffel, B. Greer, W. Van Dooren, & S. Mukhopadhyay (Eds.), *Words and worlds: Modeling verbal descriptions of situations* (pp. 245– 263). Sense Publishers.

Despina, D., & Harikleia, L. 2014. "Addition and Subtraction Word Problems in Greek Grade A and Grade B Mathematics Textbooks: Distribution and Children's Understanding". *International Journal for Mathematics Teaching and Learning,* 8: 340.

DeWindt-King, A. M., & Goldin, G. A. 2003. "Children's visual imagery: Aspects of cognitive representation in solving problems with fractions". *Mediterranean Journal for Research in Mathematics Education,* 2(1): 1-42.

Elia, I., & Philippou, G. 2004. The functions of pictures in problem solving. In M. J. Hoines & A. B. Fuglestad (Eds.), *Proceedings of the 28th conference of the International Group for the Psychology of Mathematics Education* (Vol.2, pp.327–334). Bergen University College.

Fagginger Auer, M. F., Hickendorff, M., Van Putten, C. M., Beguin, A. A., & Heiser, W. J. 2016. Multilevel latent class analysis for large-scale educational assessment data Exploring the relation between the curriculum and students' mathematical strategies. *Applied Measurement in Education*, 29(2): 144– 159. https:// doi. org/ 10. 1080/ 08957 347. 2016. 11389 59

Ferrucci, B. J., Kaur, B., Carter, J. A., & Yeap, B. H. 2008. Using a model approach to enhance algebraic thinking in the elementary school mathematics classroom. In C. E. Greenes, & R. Rubenstein (Eds.), *Algebra and algebraic thinking in school mathematics* (pp. 195–209). National Council of Teachers of Mathematics.

Greer, B. 1992. Multiplication and division as models of situations. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 276–295). Macmillan Publishing Co, Inc.

Hegarty, M., & Kozhevnikov, M. 1999. "Types of visual–spatial representations and mathematical problem solving". *Journal of Educational Psychology,* 91(4): 684–689. https:// doi. org/ 10. 1037/ 0022- 0663. 91.4. 684

Hegarty, M., Mayer, R. E., & Monk, C. A. 1995. "Comprehension of arithmetic word problems: a comparison of successful and unsuccessful problem solvers". *Journal of Educational Psychology,* 87(1): 18–32. https:// doi. org/ 10. 1037/ 0022- 0663. 87.1. 18

Heinze, A., Marschick, F., & Lipowsky, F. 2009. "Addition and subtraction of three-digit numbers. Adaptive strategy use and the influence of instruction in German third grade". *ZDM – The International Journal on Mathematics Education,* 41: 591–604. http://dx.doi.org/10.1007/s11858-009-0205-5

Heller, J. I., & Greeno, J. G. 1978, May. Semantic processing in arithmetic word problem solving. In A*nnual meeting of the Midwestern Psychological Association, Chicago.*

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J., Chui, A. M., Wearne, D., Smith, M., Kersting, N., Manaster, A., Tseng, E., Etterbeek, W., Manaster, C., Gonzales, P., & Stigler, J. 2003. *Teaching mathematics in seven countries.* Results from the TIMSS 1999 Video Study', National Center for Education Statistics (NCES)

Kaur, B. 2019. "The why, what and how of the 'Model' method: a tool for representing and visualising relationships when solving whole number arithmetic word problems". *ZDM-Mathematics Education,* 51(1): 151–168. https:// doi. org/ 10. 1007/ s11858- 018- 1000-y

Kho, T. H. 1987. Mathematical models for solving arithmetic problems. *Proceedings of the Fourth Southeast Asian Conference on Mathematical Education (ICMI-SEAMS)* (pp.345–351). Institute of Education of Singapore.

Marshall, S. P. 2012. Schema-Based Instruction. In N. M. Seel (Ed.), *Encyclopedia of the Sciences of Learning.* Springer Sciences & Business Media. https:// doi. org/ 10. 1007/ 978-1- 4419- 1428-6_261

Marton, F. 2015. *Necessary conditions of learning.* Routledge.

Mayer, R. E., Sims, V., & Tajika, H. 1995. "A comparison of how textbooks teach mathematical problem solving in Japan and the United States". *American Educational Research Journal,* 32(2): 443– 460. https:// doi. org/ 10. 2307/ 11634 38

Mullis, I. V. S., Martin, M. O., Foy, P., Kelly, D. L., & Fishbein, B. 2020. *TIMSS 2019 International Results in Mathematics and Science.* Retrieved from Boston College, TIMSS and PIRLS International Study Center. Retrieved 20 June, 2022 from https://www. timssandpirls.bc.edu/timss2019/ international-results/

Novotná J., & Chvál M. 2018. "Impact of order of data in word problems on division of a whole into unequal parts". *Journal on Efficiency and Responsibility in Education and Science,* 11 (4): 85-92. https://doi.org/10.7160/eriesj.2018.110403

Orrantia, J., Gonzalez, L. B., & Vicente, S. 2005. "Analysing arithmetic word problems in Primary Education textbooks". *Journal for the Study of Education and Development,* 28(4): 429–451.

Petersson, J., Sayers, J., Rosenqvist, E., & Andrews, P. 2021. "Two novel approaches to the content analysis of school mathematics textbooks". *International Journal of Research & Method in Education,* 44(2): 208–222. https:// doi. org/ 10. 1080/ 17437 27X. 2020. 17664 37

Polya, G. 1945. *How to solve it?.* Princeton University Press.

Rao, N., Ng, S. S. N., & Pearson, E. 2010. Preschool pedagogy: a fusion of traditional Chinese beliefs and contemporary notions of appropriate practice. In C. Chan & N. Rao (Eds.), *Revisiting the Chinese learner. CERC studies in comparative education* (pp. 255–279). Dordrecht: Springer. https:// doi. org/ 10. 1007/ 978- 90- 481- 3840-1_9

Rezat, S. 2006. A model of textbook use. In J. Novotna, H. Moraova, M. Kratka, & N. StehUkova, (Eds.), *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol.4, pp.409–416). PME.

Rezat, S., & Strässer, R. 2014. Mathematics textbooks and how they are used. In P. Andrews, & T. Rowland (Eds.), *Master class in mathematics education: International perspectives on teaching*

and learning (pp. 51–62). New York (NY: Bloomsbury.
Riley, M. S., & Greeno, J. G. 1988. "Developmental analysis of understanding language about quantities of solving problems". *Cognition & Instruction,* 5(1): 49–101. https:// doi. org/ 10. 1207/ s1532 690xc i0501_2
Schmidt, W. H., McKnight, C. C., Houang, R. T., Wang, H. A., Wiley, D. E., Cogan, L. S., & Wolfe, R. G. 2001. *Why schools matter: A cross-national comparison of curriculum and learning*. The Jossey- Bass Education Series.
Schmidt, W. H., McKnight, C. C., Valverde, G. A., Houang, R. T., & Wiley, D. E. 1997. *Many visions, many aims: A cross-national investigation of curricular intentions in school mathematics* (vol. 1). Kluwer.
Schoen, R. C., Champagne, Z., Whitacre, I., & McCrackin, S. 2021. "Comparing the frequency and variation of additive word problems in United States first-grade textbooks in the 1980s and the Common Core era". *School Science and Mathematics,* 121(2): 110–121. https:// doi. org/ 10. 1111/ ssm. 12447
Schoenfeld, A. H. 1991. On mathematics as sense-making: An informal attack on the unfortunate divorce of formal and informal mathematics. In J. F. Voss, D. N. Perkins, & J. W. Segal (Eds.), *Informal reasoning and education* (pp. 311–343). Lawrence Erlbaum Associates.
Schoenfeld, A. H. 1992. Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.)*, Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 334–370). Macmillan Publishing Co, Inc.
Siegler, R. S., & Oppenzato, C. O. 2021. Missing input: How imbalanced distributions of textbook problems affect mathematics learning. *Child Development Perspectives, 15*(2), 76–82. https:// doi. org/ 10. 1111/ cdep. 12402
Sievert, H., van den Ham, A. K., & Heinze, A. 2021. Are first graders' arithmetic skills related to the quality of mathematics textbooks? A study on students' use of arithmetic principles. *Learning and Instruction, 71,* 101401. https:// doi. org/ 10. 1016/j. learn instr uc. 2020. 101401
Sievert, H., van den Ham, A.K., Niedermeyer, I., & Heinze, A. 2019. "Effects of mathematics textbooks on the development of primary school children's adaptive expertise in arithmetic". *Learning and Individual Differences,* 74: 101716. https://doi.org/10.1016/j.lindif.2019.02.006
Stigler, J. W., Fuson, K. C., Ham, M., & Kim, M. S. 1986. "An analysis of addition and subtraction word problems in American and Soviet elementary mathematics textbooks". *Cognition and Instruction,* 3(3): 153–171. https:// doi. org/ 10. 1207/ s1532 690xc i0303_1
Tárraga, R., Tarin, J., & Lacruz, I. 2021. "Analysis of word problems in primary education mathematics textbooks in Spain". *Mathematics,* 9(17): 2123. https:// doi. org/ 10. 3390/ math9 172123
Törnroos, J. 2005. "Mathematics Textbooks, opportunity to learn and student achievement". *Studies in Educational Evaluation,* 31(4): 315–327. https:// doi. org/ 10. 1016/j. stued uc. 2005. 11. 005
Vergnaud, G. 1991. *El nino, las Matematicas y la realidad* [Child, Mathematics and reality]. Trillas.
Verschaffel, L., De Corte, E., & Pauwels, A. 1992. Solving compare problems: An eye movement test of Lewis and Mayer's consistency hypothesis. *Journal of Educational Psychology,* 84(1): 85–94. https:// doi. org/ 10. 1037/ 0022- 0663. 84.1. 85
Verschaffel, L., Depaepe, F., & Van Dooren, W. 2020. Word problems in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 908–911). Springer.
Verschaffel, L., Greer, B., & De Corte, E. 2000. *Making sense of word problems*. Swets & Zeitlinger Publishers.

Verschaffel, L., Greer, B., & De Corte, E. 2007. Whole number concepts and operations. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 557–628). Information Age Publishing Inc.

Vicente, S., Manchado, E., & Verschaffel, L. 2018. "Solving arithmetic word problems. An analysis of Spanish textbooks". *Culture and Education,* 30(1): 71–104. https:// doi. org/ 10. 1080/ 11356 405. 2017.14216 06

Vicente, S., Orrantia, J., & Verschaffel, L. 2008. "Influence of mathematical and situational knowledge on arithmetic word problem solving: Textual and graphical aids". *Journal for the Study of Education and Development,* 31(4): 463–483. https:// doi. org/ 10. 1174/ 02103 70087 86140 959

Vicente, S., Sánchez, R., & Verschaffel, L. 2020. "Word problem solving approaches in mathematics textbooks: a comparison between Singapore and Spain". *European Journal of Psychology of Education,* 35: 567–587. https://doi.org/10.1007/s10212-019-00447-3

Vicente, S., Verschaffel, L., Sánchez, R., & Sánchez, D. 2022. "Arithmetic word problem solving. Analysis of Singaporean and Spanish textbooks". *Educ Stud Math,* 111: 375–397. https://doi.org/10.1007/s10649-022-10169-x

Xin, Y.P. 2019. The effect of a conceptual model-based approach on 'additive' word problem solving of elementary students struggling in mathematics. *ZDM-Mathematics Education, 51*: 139–150. https:// doi. org/ 10. 1007/ s11858- 018- 1002-9