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Designing a Heuristic Based on Flipped Classroom Approaches Aligned with the 5E Instructional Model to Teach Mathematics

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Abstract

This study was conducted to assess the efficacy of a design heuristic to assist teachers in devising flipped classroom (FC) lesson plans based on the 5E instructional model. Data comprised lesson plans collected from 121 middle mathematics teachers in Saudi Arabia who used the design heuristic during a four week online professional development course. A document analysis was performed to examine how the teachers utilised the design heuristic. This led to the identification of three principal categories: Pre out-of-class phases to stimulate students, in-class phases comprising student-centred learning activities, and post out-of-class phases to consolidate what was learnt. The lesson plans were then analysed using the 5E model scoring instrument. The results revealed that teachers planned to use Pre out-of-class phases for engagement, post out-of-class phases for consolidation, and in-class phases principally for carrying out student-centred activities. Overall, the analysis revealed that middle mathematics teachers had the ability to design FC lesson plans that were largely aligned with all five phases of the 5E instructional model. Based on the findings, the researcher recommends the use of flipped classroom approaches aligned with the 5E instructional model in different subjects and at various stages of education. This study also concludes with recommendations regarding future research in this area.

Keywords: *Flipped classroom; 5E instructional model; inquiry-based learning; mathematics education; technology-enhanced learning; design heuristic.*

Introduction

Over the past ten years, teacher professional development courses aimed at promoting FC approaches in mathematics education have become increasingly pertinent and well-received. By attending such courses, teachers primarily learn conventional methods of flipping classes, where information is transmitted before class via instructional videos and the additional in-class time made available is spent engaging in a range of student-centred learning activities (Wasserman et al., 2015). To ensure FC scenarios are as effective as possible, a recent meta-analysis by Zheng et al. (2020) advises integrating FC approaches with pedagogical models such as inquiry-based learning. The latter is ideal for implementation in FC scenarios during mathematics education because the in-class time made available can be utilised to allow students to hypothesise, investigate, convey, and justify solutions to problems (Love et al., 2015).

Song and Kapur (2017) are among the scholars who initially developed and modified FC approaches to facilitate learning through inquiry in flipped scenarios. They integrated FC with the pedagogy of productive failure (Kapur, 2010) to create a strategy they labelled productive

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failure-based FC. Akin to the 5E model (Bybee, 2014), Song and Kapur incorporated engagement, exploration, and explanation phases into their approach. Predicated on the constructivist approach to learning, the premise of the 5E Instructional Model (Bybee, 2014) is that learning is optimised by allowing students to strive to understand concepts and problems by themselves, with the teacher's role limited to that of a guide. The outsourced transmission of information that occurs as a result of combining FC scenarios with the 5E model frees up additional in-class time for student-centred activities. Thus far, 5E-based FC settings have been explored in subjects as diverse as ihistory education (Lo, 2017), physics (Aşıksoy & Ozdamli, 2017), and biology (Jensen, Kummer & Godoy, 2015).

Moreover, according to Cevikbas and Kaiser (2020), almost all studies conducted in the FC context have concentrated on students and what they learn in mathematics classrooms rather than teachers and teaching (e.g. Antonio, 2022; Günbatar, 2021a, b; Jia et al., 2021; Kay et al., 2019; Kim, 2017; Kim et al., 2014; Le Roux & Nagel, 2018; Stover & Houston, 2019; Yılmaz, 2020; Özüdoğru, 2022). In the current study, the researcher used a design heuristic (Schallert et al., 2022) to assist teachers in planning lessons for FC scenarios that are inquiry-based. The proposal was to combine the FC model with the constructivist-based 5E instructional model, which comprises five phases: engage, explore, explain, elaborate, and evaluate (Bybee, 2014). The entire inquiry cycle must be covered in a 5E-based FC scenario. Thus, the 5E-based FC presented in this paper can be viewed as an extension of the productive failure-based approach FC. The two questions guiding this research were as follows:

(1) To what extent does the presented design heuristic support middle mathematics teachers in developing FC lesson plans that align with the 5E instructional model?
(2) How do middle mathematics teachers adopt the presented design heuristic in their lesson plans to facilitate the use and organisation of in-class and out-of-class phases in FC scenarios?

Theoretical Background

Flipped Classroom Approaches

Premised on a hybrid form of teaching that reverses traditional teaching methods by focusing on students rather than teachers, the flipped classroom (FC) is a pioneering approach to teaching that involves implementing learning activities inside the classroom and delivering lectures outside the classroom (Bergmann & Sams, 2012; Erbas & Yenmez, 2011; Long et al., 2017; Cevikbas & Kaiser, 2020). It is defined by Abeysekera and Dawson (2015) as an approach whereby the content taught is principally transmitted outside the classroom; the additional in-class time that results is used to engage in student-centred learning activities, and to accrue full benefits from in-class work, students must complete both pre- and post-class activities. Weller and Cooper (1985) reported that a useful approach for students is to learn from worked-out examples. Because there is no need to limit the implementation of information transmission phases of a FC prior to class, the researcher based the design heuristic on this definition. The freedom to outsource direct instruction is essential in facilitating learning through inquiry in FC scenarios.

Despite early attempts to define FC in the early 2000s (Baker, 2000; Lage et al., 2000), an agreed definition is yet to be reached. Nevertheless, researchers and educators alike concur that FC is a mode of pedagogy that is student-centred and that by making additional class hours available for social interaction, working together, inquiry, and in-depth learning, it is likely to enhance the calibre of both teaching and learning (Cevikbas & Argün, 2017; Cevikbas & Kaiser, 2020, 2021).

Preliminary definitions of FC alluded to it as schoolwork at home and homework at school (Baker, 2000; Bergmann & Sams, 2012; Lage et al., 2000), whilst later definitions extended this conceptualisation (e.g., Bishop & Verleger, 2013; FLN, 2014). For instance, Bishop and Verleger (2013) defined FC as comprising two elements: computer-based individual learning activities outside the classroom and interactive group learning activities inside the classroom. This indicates that lecture and explanatory videos constitute a compulsory feature of out-of-class activities, and are supplemented by additional resources such as slides, podcasts, lecture notes, and articles. Alternative ways in which flipping teaching has been described are increasing social interactions, conducting quizzes, hosting question-and-answer sessions, and establishing an environment both inside and outside the classroom that is enriched by technology (Abeysekera & Dawson, 2015; Cevikbas, 2018; Cevikbas & Kaiser, 2020, 2021). Additional ways in which FC can be applied are for students to watch videos or use other learning resources inside the classroom (as opposed to outside) (Howitt & Pegrum, 2015), or to make the use of videos discretionary (Bergmann & Sams, 2012).

Affordances of a FC in Teaching Mathematics

In contrast to students in the late twentieth century, the students of today exhibit notably different attributes, expectations, and inclinations (Cevikbas & Argün, 2017). For instance, their preference is to gain rapid access to information, and to create their own knowledge in an enjoyable way by utilising an array of digital technology channels (Cevikbas, 2018; Engelbrecht et al., 2020). To address these changes, adaptations are required to both learning environments and instructional methods. Stressing the inherent value of these pedagogical applications, the principles and standards of school mathematics have been issued in the USA by the National Council of Teachers of Mathematics (NCTM (2008). By transposing conventional approaches to teaching using technology, FC furnishes students with a bespoke learning environment. In so doing, it assists in creating high-quality teaching activities in the domain of mathematics (Chen & Wen, 2019) and enhances the learning opportunities available to mathematics students. Specifically, FC helps to clarify the objectives of collaborative learning and stimulates students to improve their ability to think critically and ruminate on mathematics problems prior to engaging in classroom activities (Branford, 1908; Mazur et al., 2015; Voigt et al., 2020). Within classrooms, teachers also have extra time in which to implement problem-solving activities, inquiry-based activities, hands-on activities, and in-depth analysis (Schmidt & Ralph, 2016). In such a scenario, the teacher's role is akin to that of a coach or facilitator, rather than simply providing students with the requisite knowledge (Hwang et al., 2015). They have the capacity to adopt a creative yet strategic approach to their teaching, and can make sense of the mathematical thinking employed by students (Fulton, 2012). In so doing, the FC approach converts mathematics classrooms into environments for analytical thinking, experimental inquiry, and creating linkages with other domains of STEM (Bergmann & Sams, 2012).

Using FC, teachers benefit as it assists them in enhancing their professional competencies and transmuting classroom dynamics, whilst students benefit by being motivated to partake in mathematics from cognitive, emotional, and behavioural perspectives (Cevikbas & Argün, 2017). Such innovative transformation of teaching experiences will engender compelling ideas and give rise to a genuine vision (Brown, 2018). The features of FC are such that even if there is no direct need to possess advanced mastery of technology, teachers must, as a minimum, possess fundamental skills in using technology to teach mathematics. Based on this, they can learn novel technologies and pedagogical techniques (Brown, 2018). The role of teachers in FCs is an active one in that they provide their students with scaffolding and guidance when they require support from qualified professionals (Cevikbas & Argün, 2017; FLN, 2014). Moreover, they also monitor the progress of students and provide prompt and detailed feedback. FCs also ameliorate disciplinary issues in the classroom (Cockrum, 2014), enhance teacher–student interactions (Bergmann & Sams 2012; Brown 2018; Cevikbas & Argün, 2017; Lo & Hew, 2017), and transform classroom management and ensure greater transparency by enabling parents to follow the activities in which teachers and students are engaging (Bergmann & Sams, 2012).

Difficulties of Teaching in Flipped Mathematics Classrooms

In spite of the myriad benefits of using FCs in mathematics teaching, empirical studies have also reported some of the challenges teachers may face, which can be grouped into three broad categories: (1) paradigm shift, (2) content, and (3) technical requirements. The paradigm shift alludes to altering the pedagogical paradigm of teaching and learning mathematics (Cevikbas & Argün, 2017; Lo & Hew, 2017). Paradigmatic obstacles to flipped teaching and learning may arise as a result of the beliefs and perceptions held by students and teachers. For instance, because students do not assume responsibility for independent learning, they may fail to complete the out-of-class tasks and thus attend class without having seen the lecture videos. Another issue that arises concerns the need for teachers to create content that is subject-specific. To effectively deliver flipped teaching, lecture videos, notes, slides, and teaching materials all need to be thoroughly prepared (Chen, 2016; Lo & Hew, 2017). Moreover, locating customised lecture videos or other forms of content that fulfil the needs and expectations of teachers and students is immensely challenging. Even though numerous videos can be accessed on online platforms such as YouTube, Teacher Tube, and Khan Academy, they may not cover or align with all the topics taught in school mathematics (Chen, 2016). Consequently, teachers must devise their own lecture videos, a task that takes an inordinate amount of time. An additional challenge that arises in implementing FC is ensuring teachers possess the technical means to teach and learn mathematics. The fundamental framework of FCs may be adversely affected by technical issues related to accessing the Internet and mobile devices, an issue that cannot be overstated. FC practices function poorly without the Internet, and teachers need to have the capacity to strategically utilise technology when teaching mathematics. Teachers who need to improve their competencies in using technology may face complex difficulties in concomitantly organising and managing tasks, resources, students, and knowledge (Trigueros et al., 2020). Furthermore, flipping the classes may give rise to issues such as unwillingness, dissatisfaction, and opposition to FC (Bagley, 2020; Chen, 2016).

The 5E Instructional Model

The basis of the 5E model lies in constructivist learning theories such as social constructivism (Vygotsky, 1978) and Dewey's reflective thinking (1938). Because the construction of knowledge is based on what is previously known, it is vital to extract such knowledge (Borko & Putnam, 1996). The 5E Instructional Model aims to ensure students actively engage in the process of teaching and learning. and comprises five distinct phases; engage, explore, explain, elaborate, as presented in Figure 1. Within the 5E model (Bybee, 2014), a learning cycle commences with the engagement phase, whereby teachers may present a problematic scenario, define a problem, or devise questions to be investigated in conjunction with the students. Incorporated in this phase is the engagement of students with a question that has a scientific orientation. The activities taking place in this phase should stimulate students and allow them to make links to previous experiences. In the second phase, teachers facilitate a process of exploration and inquiry among students. This process has two fundamental components: students prioritising evidence when addressing questions, and students devising explanations from such evidence. Notably, Harrison (2014) found that in inquiry-based learning scenarios, teachers typically assess students' learning by observing what they do. In the third phase, explanation, teachers support learners in explaining their findings by, for example, helping them find suitable terms or concepts. The role of the teacher is to generate questions that direct students to analyse, build, clarify, and adjust their comprehension of scientific phenomena (Jackson et al., 2020). Two additional yet vital components of such inquiry are to encourage students to connect their explanation to scientific knowledge, and to communicate and provide a rationale for this explanation. In the next phase, elaboration, teachers strive to engage students in activities that enable them to generalise concepts, processes. or abilities by applying what they have learnt to similar but novel scenarios., In so doing, students prioritise evidence when responding to questions and derive explanations from such evidence. In the final phase, evaluation, learners assess their comprehension by utilising the skills they have gained whereby the essential inquiry feature students communicate and justify explanations could occur again. The role of the teacher in this process is to provide learners with feedback on their achievement in the formative assessments (Jackson et al., 2020).

Figure 1. The 5E Inquiry Model.

Based on the fact that the five vital aspects of classroom inquiry (National Research Council, 2000) can be incorporated into 5E lessons, as detailed in Table 1, the 5E model was used by the researcher to develop the design heuristic. These five aspects are as follows: (i) engaging students in questions that are oriented towards science, (ii) prioritising evidence when responding to questions, (iii) deriving explanations from such evidence, (iv) linking such explanations to scientific knowledge. and (v) conveying and providing a justification for these explanations (National Research Council, 2000, p. 29). Extensive research has been conducted on the degree to which the 5E model can be applied. It was initially created for science education, although its application in other subjects such as mathematics had already been investigated in earlier studies (Omotayo & Adeleke, 2017; Tuna & Kacar, 2013), yielding positive outcomes. For instance, multiple studies (e.g. Bilgin et al., 2013) have found that applying the 5E model in STEM education can exert a positive impact on student achievement.

The 5E-Based Flipped Classroom Approach

With respect to STEM education, Aşıksoy and Ozdamli (2017) employed a mixed-method approach to investigate a 5E-based FC on a physics course in which 94 engineering students were involved. First, a randomised control trial was conducted which revealed significantly higher levels of student achievement in the 5E-based FC group compared to the control group, for whom the 5E model only was applied. Second, semi-structured interviews were conducted to elicit the perceptions of students, the findings of which revealed that students found learning in the 5E-based FC to have greater meaning. In addition, it enabled them to better understand the link between physics principles and real life as they were given more time to discuss this association during class. However, different results were obtained from a quasi-experimental study (Jensen et al., 2015) examining the effect of the 5E-based FC on 108 students engaged in biology education. Specifically, no significant differences were found between the flipped 5E class and the non-flipped 5E class with respect to levels of satisfaction and achievement among the students. Moreover, the researchers concluded that in both conditions, learning gains were attributable to the active learning approach employed and not flipping the class.

A further advancement of conventional FC approaches towards a 5E-based FC takes the form of the productive failure-based FC in mathematics education (Kerrigan, 2018; Song & Kapur, 2017). In this approach, learners begin by addressing problems in-class with a teacher available to support them should their strategies fail. They then view an explanatory video at home to integrate their findings. Thus, a productive failure-based FC (Song & Kapur, 2017) covers the first three phases of the 5E model only, that is, engagement, exploration, and explanation. Adopting a quasi-experimental design, Song and Kapur (2017) compared the productive failure-based FC with the traditional FC, where students are given direct instruction in the form of homework and then apply what they have learnt in class. The results revealed that the conceptual understanding of mathematics students was positively impacted by the use of the productive failure-based FC. However, in a 5E-based FC scenario, it is essential to cover all five phases of the entire inquiry cycle. Thus, the 5E-based FC described in this paper can be viewed as an advancement of the productive failure-based FC.

Design Heuristic For 5E-Based Flipped Classroom Scenarios

The researcher developed a design heuristic based on the 5E instructional model to help teachers devise lesson plans for inquiry-based FC situations (Bybee, 2009). This design heuristic (see Table 1) describes the activities that take place in the classroom for each phase of the model, along with those that can take place outside in the form of homework. In accordance with the definition of a FC provided by Abeysekera and Dawson (2015), the researcher provided opportunities for students to engage in activities that principally involved direct instruction (in the form of homework) along with student-centred activities taking place in the classroom. Incorporated into the design heuristic were examples of digital educational materials, but these should only be used if they advance students' learning. Thus, the design heuristic serves as an approximate blueprint for teachers to follow when devising lesson plans appropriate for 5E-based FC scenarios. This affords teachers the flexibility to modify the heuristic in accordance with their teaching requirements, and hence they are not compelled to include both homework activities and in-class activities for each phase of the 5E.

Table 1. Design Heuristic for 5E-Based Flipped Classroom Scenarios.

5E phase	Feature of 5E Phase	Out-of-class activities	In-class activities
Engagement	Activity to elicit prior knowledge and raise curiosity.	The teacher includes a scenario, material for students to explore, and questions to respond to (using digital material). The teacher raises questions for students to uncover prior knowledge related to the lesson without seeking any kind of correct answer. Students go through these materials at their own pace and note any questions that arise.	During the in-class session, the teacher facilitate in-depth discussions to elaborate or develop on questions to engage learners.
Exploration	Activity to investigate and inquire with teacher's guidance.	Teacher provides a learning environment to be explored. Students writing homework and explored online resources.	The role of the teacher is to guide the discussion process and to encourage involvement through mutual exploration and discovery. Students explore the learning environment and share their findings with the class.
Explanation	Teachers support learners in explaining their findings by, for example, helping them find suitable terms or concepts.	The scientific explanation of the phenomenon is highly significant. in the "explain" phase, as the scientific concept(s) or theories are made clear and comprehensible. Teacher introduces relevant concepts or theories by using video, textbook or other materials. Students study the provided material and compare it with their explanations.	The teacher highlights the key elements of the first two phases and asks the students to explain them. Building on students' explanations and previous experiences, the teacher briefly introduces the scientific concepts.
Elaboration	New related experiences are presented.	Teacher help students transfer prior knowledge to new learning contexts and experiences through challenging yet achievable activities such as using video and/or textbook materials. The students are actively engaged in new learning experiences that extend, expand, and enrich the concepts and skills that have been developed in the previous phases, but closely related situations.	Teacher promotes elaboration. Students apply the knowledge gained to solve new, but closely related problems.

Evaluation	The purpose of this phase is to evaluate students' knowledge of the Scientific phenomenon.	Teacher provides self-assessment and diagnostic assessment for students. Diagnostic assessments in education help educators understand their students' strengths, weaknesses, knowledge level, and skillset prior to beginning instruction. Students engage in self-assessment and diagnostic assessment tasks to reflect on their learning process.	Teacher applies an assessment technique to determine a complex attribute of an individual or group of individuals. This involves gathering and interpreting information about student level of attainment of learning goals.

Methodology

Participants and the Online Professional Development Course

The sample comprised one hundred and twenty-one middle mathematics teachers, all of whom were teaching students aged 13–15 years in various kinds of middle schools. These participants were taking part in an online professional development course set up by the researcher on the Edmodo, the topic of which was using the 5E-based FC approach in middle mathematics education. The researcher made the decision to apply the design heuristic in an online course due to the ease with which information can be exchanged and the opportunities for teachers to take part regardless of time and place. The implementation process took place over a period of four weeks, with 2 to 3 lessons per week. All participants completed the online course.

At the start of the course, videos and inspiring examples of lesson plans were made available by the researcher. These could also be accessed on a website. The researcher believes that such videos and lesson plans are two of the principal elements that shape the inquiry-based teaching conceptions of mathematics teachers' participating in a professional development course on assisted inquiry-based teaching. Upon completion of such a course, teachers are expected to have created teaching products, in this case final lesson plans, that reflect the process of learning.

To ensure teachers were conversant with the design heuristic and could use it to plan lessons, a number of steps were taken. The first three steps involved using videos, articles, and various activities to introduce teachers to the 5E instructional model, FC approaches, and the design heuristic. In the fourth step, the researcher presented four examples of lesson plans created using the design heuristic. Using guiding questions, the researcher then asked teachers to discuss these examples on online forums. In the fifth step, participants used the design heuristic to create a lesson plan on a topic of their choosing. Lastly, in the sixth and final step, feedback on these lesson plans was provided to participants by the online tutor and their peers.

For this study, the researcher concentrated on the lesson plans created during the fifth step of the online course. Importantly, the online tutor did not provide any support during the design phase; participants were simply asked to use the design heuristic for the purposes of lesson planning. During this process, teachers were requested to provide explicit lesson objectives that were appropriate in the sense that they aligned with the curriculum of their school. Teachers were also asked to describe as fully as possible the out-of-class and in-class activities they planned for each 5E phase along with a complete materials list.

Data Analysis

To determine how teachers implemented the design heuristic in their lesson plans, elements of a qualitative content analysis (Schreier, 2012) that formed part of a document analysis approach (Bowen, 2009) were utilised to code and analyse the one hundred and twenty-one completed lesson plans. To assess the degree to which the design heuristic supported participants in developing lesson plans that align with the 5E instructional model, the researcher employed the 5E lesson plan scoring instrument (Goldston et al., 2013).

First, the researcher performed a document analysis (Bowen, 2009) of the collected lesson plans to identify instructional patterns concerning the use and organisation of in-class and out-of-class phases. The researcher chose to analyse written lesson plans as these included comprehensive descriptions, references, and materials that were of enormous value for the current study. Furthermore, it ensured no interaction effects occurred between participants and the researcher (Kondracki et al., 2002). To analyse the data, the researcher employed aspects of a mixed deductive-inductive approach to content analysis (Schreier, 2014). Specifically, the researcher developed an initial coding list predicated on the design heuristic, including for each 5E phase an in-class and out of-class category. Following this, the initial coding list was adjusted and refined by adding new codes and rewriting categories so that they were mutually exclusive (Miles & Huberman, 1994). The focus throughout was on latent (information given indirectly) and semantic (close to participants' language) features of the data. Examples of latent codes are: 'activation of prior knowledge' and 'outsourced information transmission for consolidation'. Examples of semantic codes are: 'teacher answers questions during engagement', 'use video as homework in the explain phase', and 'students tackle similar situations'

Goldston et al. (2013) employed a psychometric approach to devise the 5E inquiry lesson plan scoring instrument version 2 for inquiry-based teaching (hereafter referred to as 5E ILPv2. A total of 224 pre-service science teachers then verified the 5E ILPv2. Factor analysis confirmed the five key factors of the 5E ILPv2: engage (0.95), explore (0.97), explain (0.93), elaborate (0.95), and evaluate (0.94). Having generated an overall reliability estimate of 0.96, the 5E ILPv2 was deemed suitable for evaluating written 5E lesson plans.

The 5E ILPv2 comprises four items for the engage phase, four items for the explore phase, six items for the explain phase, three items for the elaborate phase, and four items for the evaluation phase. It also utilises a 5-point Likert scale ranging from 0 to 4 points per item, which are scored as follows: unacceptable (score 0), poor (score 1), average (score 2), good (score 3), and excellent (score 4).

The 5E IPLv2 can be employed to measure the ability of a teacher to devise a 5E lesson plan, and also guides teacher educators in how to modify and adjust their techniques for teaching students how to create 5E lesson plans. The researcher used the 5E IPLv2 to identify both the strengths and weaknesses of the proposed design heuristic. To determine reliability, the 121 lesson plans were rated by the researcher and three experienced teacher educators in accordance with the items and scoring criteria of the 5E ILPv2. First, the four raters read the comprehensive explanation for each scoring criterion in order to acquaint themselves with the scoring instrument, and then separately scored all the lesson plans. Following this, the raters compared and discussed their scores until an agreement was obtained. Lastly, instead of using descriptive statistics to present the results, the researcher clustered the scoring criteria into 'fulfilled items' and 'failed items'. The former comprised lesson plans that received a score from 2 to 4 ('average' to 'excellent'), whilst the latter comprised lesson plans that received a score lower than 2 ('unacceptable' or 'poor').

Results and Discussion

The First Research Question Stated: To what extent does the presented design heuristic support middle mathematics teachers in developing FC lesson plans that align with the 5E instructional model?

In accordance with the 5E ILPv2 (Goldston et al., 2013), the results of the analysis revealed that all 121 collected lesson plans aligned with the engagement, exploration, explanation, elaboration and evaluation phases of the 5E instructional model (see Table 2).

As presented in Table 1, 98% (119 teachers) of the lesson plans adequately addressed engage item 1. As noted previously, extracting prior knowledge is essential as the construction of knowledge is reliant on this (Borko & Putnam, 1996). Following the first item, the researcher changed nothing in the design heuristic the activation of prior knowledge in the engagement's out-of-class phase. This is because, in the engagement phase. virtually every participant aimed to use homework assignments before class to extract prior knowledge from students, enabling participants to carry on planning to activate prior knowledge in the second, third, and fourth items, with scores of 92%, 95%, and 94%, respectively.

With respect to 'explore' items 1, 2, 3 and 4, virtually every participant strived to address all items to a satisfactory degree in their lesson plans. This is evidenced by the fact that the learning activities they proposed to implement were hands-on and student-centred, which is typical of in-class activities in FC approaches (Long et al., 2017). Moreover, based on the constructivist approach to learning, the 5E Instructional Model (Bybee, 2014) posits that optimal learning is achieved by allowing students to understand problems by themselves, with teachers serving only as a guide, as contemporary students exhibit a preference for accessing information rapidly and wish to construct their own knowledge in an enjoyable manner, principally by using a wide array of digital technology channels (Cevikbas, 2018; Engelbrecht et al., 2020). Also of note is the fact that most participants continued to concentrate on evaluating students' learning during both the explanation phase and the exploration phase.

With respect to the explanation phase, the six items were addressed in almost all the lesson plans. This is evidenced by the fact that the vast majority of participants aimed to provide a full explanation of the skills or concepts they expected students to acquire in a post out-of-class phase. This result appears to align with the category 'post out-of-class phases for consolidation', which was created during this study through document analysis. That said, a number of teachers simply aimed to provide a video for consolidation as homework, which appears to align with the productive failure-based FC approach (Kerrigan, 2018).

In the elaborate phase, it was notable that nearly every participant strived to address the first three items. Regarding item 1, Goldston et al. (2013) included this along with 'explain' item 1 to establish a clear linkage between the various 5E phases. In the elaborate phase, all teachers aimed to expose learners to novel scenarios. Accordingly, the elaborate activities in all lesson plans were created to motivate learners to make real-world connections with the knowledge acquired (see Table 3, elaborate item 3). This aligns with the findings of Aşıksoy and Ozdamli (2017), whose participants stated that the 5E-based FC approach enabled them to better understand the links between the knowledge they acquired and real-world scenarios as there was more time available in class to consider such links.

In the evaluation phase, all lesson plans sufficiently addressed items 1–4. The researcher attributes this to the fact that various examples of assessment tools were included in this phase to stimulate teachers, hence they did not find it difficult to select a suitable assessment strategy. The role of the teacher in this stage is to furnish students with feedback on their performance in formative assessments (Jackson et al., 2020). Such findings confirm that for teachers, assessing learning through inquiry represents a highly effective tool. Moreover, it is strongly related to the 5Emodel, the applicability of which in subjects such as mathematics has already been investigated in previous studies (Omotayo & Adeleke, 2017; Tuna & Kacar, 2013; Bilgin et al., 2013), yielding positive outcomes. The findings are therefore not in accord with those of Harrison (2014) who reported that in inquiry-based learning scenarios, teachers principally concentrate on assessing learning during the exploration phase by observing students.

Table 2. Evaluation of Lesson Plans According to the 5E Instructional Model.

5E ILPv2 item	Item description	Number of lesson plans that fulfilled the criterion (N = 121)
Engage item 1	The engage phase elicits students' prior knowledge (based upon the objectives).	98%
Engage item 2	The engage phase increases student interest/motivation to learn.	92%
Engage item 3	The engage phase provides opportunities for student discussion/questions (or invites such questions).	95%
Engage item 4	The engage phase leads to the exploration phase.	94%
Explore item 1	During the exploration phase, teachers present instructions.	90%
Explore item 2	Learning activities in the exploration phase involve hands-on/minds-on activities.	89%
Explore item 3	Learning activities in the exploration phase are student-centred.	85%
Explore item 4	The inquiry activities of the exploration phase reveal evidence of student learning (formative authentic assessment).	92%
Explain item 1	There is a logical transition from the exploration phase to the explanation phase.	94%
Explain item 2	The explanation phase includes teacher questions that facilitate the development of concepts and skills.	96%
Explain item 3	The explanation phase includes a mix of divergent and convergent questions for interactive discussion facilitated by the teacher and/or students to develop concepts or skills.	95%
Explain item 4	The explanation phase includes a complete explanation of the concept(s) and/or skill(s) taught.	98%
Explain item 5	The explanation phase provides a variety of approaches to explaining and illustrating a concept or skill.	93%
Explain item 6	The discussions or activity taking place during the explanation phase enable the teacher to assess students' present understanding of concept(s) or skill(s).	90%
Elaborate item 1	There is a logical transition from the explanation phase to the elaboration phase.	96%
Elaborate item 2	The elaboration activities provide students with the opportunity to apply the newly acquired concepts and skills in novel areas.	89%
Elaborate item 3	The elaboration activities encourage students to identify real-life connections with the newly acquired concepts or skills.	94%
Evaluation item 1	The lesson includes summative evaluation, which may consist of a diverse array of forms and approaches.	91%

Evaluation item 2	The evaluation matches the objectives.	93%
Evaluation item 3	The evaluation criteria are clear and appropriate.	95%
Evaluation item 4	The evaluation criteria are measurable (i.e. rubrics are used).	92%

The Second Research Question Stated: How do middle mathematics teachers adopt the presented design heuristic in their lesson plans to facilitate the use and organisation of in-class and out-of-class phases in FC scenarios?

To answer this question, three main categories concerning the utilisation of the 5E-based FC design heuristic in teachers' lesson plans were created by the researcher: (a) pre out-of-class phases to engage students, (b) in-class phases to support student-centred learning activities, and (c) post out-of-class phases for consolidation.

The first category was based upon continued embedding in the engage and elaborate phases of pre out-of-class phases. The word 'engage' refers not only to the 5E phase but also the aim of engaging students in a pre-class phase. Furthermore, as preparation for the evaluation phase, several teachers planned to furnish learners with self-assessment activities as part of an initial out-of-class phase.

To make a distinction between the pre-lesson phases in this category from those employed in conventional FC scenarios, two contrasting lesson plans are presented as examples. In accordance with the 5E-based FC approach, in the first lesson plan, online resources in a pre-lesson phase were used to activate prior knowledge regarding the volume of a prism and cylinder (see Table 2). Specifically, students were required to use a dynamic geometry environment to help them independently derive the formula for the total surface area of a volume of prism and cylinder. This aligns with the approach taken in previous research (Kerrigan, 2018; Song & Kapur, 2017).

In the second lesson plan, the aim of the teacher was to a use a video to simplify algebraic expressions and write two-step equations before commencement of the class. Next, learners needed to understand the simplified algebraic expressions, solve and write two-step equations for homework using the video, and then apply the knowledge they have acquired in a follow-up lesson. This lesson plan presented a more conventional approach to flipping a class in that the novel concepts were introduced immediately as part of a pre-class phase. This is also typical of activities in FC approaches, where learners must complete both pre- and post-class activities to extract the maximum benefit from in-class work, and the newly available in-class time is utilised specifically for student-centred learning activities (Abeysekera and Dawson, 2015; Bergmann & Sams, 2012; Cevikbas & Kaiser, 2020). The pre out-of-class phases depicted in this category extract existing knowledge from students. Therefore, it can be presumed that the central notion underpinning the first category is to interpret learning as the process of restructuring and building upon knowledge that already exists. Given the mental engagement this requires on the part of learners, this core notion may be associated with existing findings in the domain of brain-based research (Jensen, 2008).

The second category concerns how the in-class time made available for student-centred learning activities in a FC is actually used. Unlike lecture-based approaches to teaching, learners in inquiry-based FC scenarios are situated at the core of the education process. As captured by the other two categories, information transmission in the lesson plans was primarily outsourced to homework phases. A number of examples from the data are now presented to illustrate this category. Overall, participants envisaged using freed-up in-class time to motivate learners to develop questions that can be investigated, either alone or in collaboration with fellow students.

As per the 5E model (Bybee, 2014), a learning cycle begins with the process of engagement whereby teachers present a problem scenario, define a problem, or devise questions to investigate in conjunction with the students. The role of the teacher is to generate questions that assist students in analysing, building, clarifying, and refining their comprehension of a scientific phenomenon (Jackson et al., 2020). Teachers are able to make sense of the mathematical thinking exhibited by students and ensure that class hours are used in both a strategic and creative manner (Fulton, 2012). Despite one or two deviations, the aim of most teachers was to allow students to investigate novel concepts whilst in class and ensure they had sufficient time to generate their findings. The explanations given by students would then form the basis for whole class discussions. In the majority of lesson plans, hands-on activities were included that required learners to apply the conceptual knowledge and skills they had gained to novel scenarios as part of the elaborate phase. This reflects the fact that a realistic vision and convincing ideas are the fruitful outcomes of dramatic transformations in the experiences of teaching (Brown, 2018).

The central concept underpinning this category appears to be aligned with constructivism, as the 5E model is rooted in constructivist learning theories such as Dewey's reflective thinking (1938). These theories posit that learners do not passively receive facts and concepts but instead are active participants in the construction of knowledge. A unique role is played in this regard by the notion of social constructivism (Vygotsky, 1978), as knowledge is created in inquiry-based FC scenarios by a group of learners. albeit with the support of teachers who serve as coaches or facilitators rather than providers of knowledge (Hwang et al., 2015), as also occurs in 5E lessons.

Regarding the third category, this proposes an alternative method for carrying out a homework assignment. In contrast to the first category, it proposes the adoption of an out-of-class phase following the main class. The ideas that underpin this category are as follows. To introduce a novel topic in FC scenarios, it is not always necessary for teachers to deliver direct instruction prior to a class. It is important to give learners the opportunity to feel that they need an introduction, or at the very minimum be grateful for the advantage this provides when it is explained by the teacher. Such a principle was originally developed by Branford (1908) and can also be applied to FC teaching, where it is important to avoid continually utilising information transmission at the start of the learning process. As far as possible, learners need to be able to experience concepts for themselves prior to them being explained by teachers. In this type of FC scenario, mistakes should be viewed as opportunities. This form of flipping a classroom is therefore described as a productive failure-based FC approach, in that learners address various problems during class and then consolidate what they have found at home (Kerrigan, 2018).

With respect to the latter, the lesson plans revealed that the majority of participants aimed to provide students with written worked-out examples or videos containing full explanations as a homework task in the explanation or elaboration phases. Moreover, a large number of teachers included one to seven minute long YouTube videos with integrated quiz assignments so that the process of watching these videos was an active one. This aligns with Bybee et al.'s (2014) observation that to enhance student explanations, a variety of methods and techniques are employed by teachers in 5E lessons, including videos. For instance, videos or other educational resources can be used by teachers in FC scenarios to elucidate concepts or processes which students may not have grasped. These typically form part of a homework task during the explanation phase. Sweller and Cooper (1985) also asserted that students find it helpful to learn from worked-out examples. When presented in a video, it is possible to provide a step-by-step solution accompanied by verbal explanations, creating what is known as an illustrative video

(Kerrigan, 2018; Song & Kapur, 2017). Consequently, there is no requirement for students to explain to themselves the intermediate steps to a solution.

A representative lesson plan is presented in Table 3 to illuminate the results of the document analysis. Its topic is how to simplify algebraic expressions, and then solve and write two-step equations. This example was selected because it addresses all three of the main categories. For the purposes of illustration, the lesson plan is translated from German to English and the descriptions of the lesson plan are summarised. Therefore, other than the effective GeoGebra activity, the materials utilised in the lesson plan are excluded as they are in German only and were not required for the purposes of the current study.

As presented in Table 2, at the start of the sequence, the first main category 'pre out-of-class phases to engage students' is covered by eliciting the prior knowledge of students before the class begins. Students, however, do not undertake the tasks out-of-class in the elaboration phase. This might be because elements of explanation and elaboration are addressed in the same lesson; therefore, incorporate a homework assignment between these phases would simply waste time. Indeed, in most lesson plans, participants did not assign homework in every phase of the 5E model simply to align with the suggested design heuristic.

In accordance with FC approaches (Abeysekera & Dawson, 2015), in the lesson plan example presented in Table 2, information transmission is primarily outsourced. The only element implemented in the engagement phase during class is to present the educational scenario for investigation so as to stimulate active exploration in-class, which in this example takes the form of a dynamic GeoGebra activity (see Table 2). This aligns with previous research (Erbas & Yenmez, 2011) which suggests that combining dynamic geometry environments, such as GeoGebra, with student-centred inquiry can nurture students' ability to engage in conjecture, thereby improving their performance. The essential value of educational applications such as this is exemplified in the principles and standards of school mathematics issued by the National Council of Teachers of Mathematics (NCTM) in the USA (2008). This included technology as a core principle on which mathematics is premised; a principle which holds that technology is an essential tool for teaching twenty-first century mathematics and every school needs to make sure that their students have the capacity to use such technology.

In the example lesson plan, students must begin developing and justifying their explanations in groups following exploration; hence a logical transition occurs from the exploration to the explanation phase. A post out-of-class consolidation phase is implemented as part of the latter, thereby determining the third main category. In this lesson plan, the intention is to use a YouTube video for consolidation, which in this case (Mazur et al., 2015) explains and illustrates how to derive the volume of a prism and cylinder.

In this lesson plan, the second main category 'in-class phases for student-centred learning activities' is primarily situated in the exploration and elaboration phases, where hands-on activities will be conducted. For the elaboration phase, students will be asked to work in pairs to solve examples from the textbook that involve calculating the total surface area of a right circular cone.

Table 3. Lesson Plan Example: 'Formula for the Total Surface Area of a Volume of a Prism and Cylinder'.

5E phase	Out-of-class activities	In-class activities
Engagement	Prior knowledge regarding the right circular volume of a prism and cylinder to be activated using online resources (a website containing online activities).	Teacher answers questions from the homework phase as part of a classroom discussion. They then present an educational scenario where students are asked to derive the formula for the total surface area of a volume of prism and cylinder.

Exploration	-	Supported by the teacher and their peers, students independently undertake the following GeoGebra dynamic activity. Supplementing this, they share their experiences and begin formulating and justifying their findings and explanations in groups.
Explanation	Students are asked to watch a YouTube video (duration: 3:00) explaining how to derive the formula for the total surface area of the volume of prism and cylinder. Students are asked to compare their derivation with that presented in the video and note any questions that arise.	Teacher answers questions from the homework phase as part of a classroom discussion.
Elaboration	-	Students should apply the knowledge gained by solving, in pairs, several textbook examples that involve calculating the total surface area of the volume of a prism and cylinder.
Evaluation	For self-assessment and diagnostic assessment, students must calculate the total surface area of the volume of a prism and cylinder based on their self-selected values and then check their results using an online calculator.	To improve the course, a formative assessment plan was designed and implemented to gather data at regular intervals during this study. The student feedback was often deeper and more personally insightful than anticipated, hence we believe that this assessment strategy could be used to improve student learning engagement through self-reflection, whilst also incorporating the student voice into the learning and assessment process.

Conclusion

The objective of this research was to assess the use of a design heuristic created to assist mathematics teachers in creating FC lesson plans that align with the 5E instructional model. To achieve this, lesson plans were collected from 121 middle mathematics teachers who implemented the design heuristic as part of a four-week professional development course delivered online. A document analysis was carried out to examine how teachers utilised the design heuristic in their lesson plans. The results revealed that participants aimed to use pre out-of-class phases for engagement, post out-of-class phases for consolidation, and in-class phases principally to implement student-centred activities. Moreover, using the proposed design heuristic, the participants were able to devise FC lesson plans that aligned with the engage, explore, explain, elaborate, and evaluation phases of the 5E instructional model.

Further Research

1- Because the 5E instructional model is widely employed in science education, it would be useful to assess the utility of the design heuristic in this domain.
2- Replicate and extend the study to include primary and high school teachers.
3- This study focused on the mathematics, but another study could focus on other areas of the education field.
4- Carry out additional studies to determine the challenges and barriers preventing FC approaches from being used with the 5E instructional model to teach mathematics.

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